1. The region is bounded above by \( y = -x + 2 \) and below by \( y = x^2 - 2x \). See the picture below.

The line and the parabola cross at those points \( x \) where \( x^2 - 2x = -x + 2 \), or \( x^2 - x - 2 = (x - 2)(x + 1) = 0 \). It follows that the points of crossing are at \( x = -1 \) and \( x = 2 \).

Thus, the area is given by
\[
\int_{-1}^{2} \left( (-x + 2) - (x^2 - 2x) \right) \, dx = \\
\int_{-1}^{2} (-x^2 + x + 2) \, dx = \left[ -x^3/3 + x^2/2 + 2x \right]_{-1}^{2} = 9/2.
\]

2. The region is bounded above by \( y = \sqrt{9 + 3x^2} \) and below by \( y = 2x \). See the picture at the right. The graphs cross where \( 2x = \sqrt{9 + 3x^2} \), or \( 4x^2 = 9 + 3x^2 \), or \( x^2 = 9 \), or \( x = 3 \).

2(a) The volume for rotation around the \( x \)-axis is:
\[
\int_{0}^{3} \pi \left( \sqrt{9 + 3x^2} \right)^2 \, dx - \int_{0}^{3} \pi (2x)^2 \, dx = \\
\int_{0}^{3} \pi(9 + 3x^2) \, dx - \int_{0}^{3} \pi(4x^2) \, dx = \int_{0}^{3} \pi(9 - x^2) \, dx = 18\pi.
\]

2(b) The volume for rotation around the \( y \)-axis is:
\[
\int_{0}^{3} (2\pi x) \left( \sqrt{9 + 3x^2} \right) \, dx - \int_{0}^{3} (2\pi x)(2x) \, dx = \\
2\pi \left( \frac{1}{9}(9 + 3x^2)^{3/2} - \frac{2}{3}x^3 \right)_{0}^{3} \\
2\pi \left( (36)^{3/2}/9 - 2\cdot3^3/3 \right) - \left( (9)^{3/2}/9 - 2\cdot0^3/3 \right) = 6\pi.
\]

3. Take a cross section of the solid where the \( y \)-coordinate is equal to \( y \). Then, the cross section is a square with base in the \( xy \)-plane. One end of the base is at \((x, y)\) and the other at \((-x, y)\). See the picture at the right.

Thus, the base of the square has length \( 2x \). It follows that the area of the square is \( 4x^2 \). Since \( 4x^2 + y^2 = 4 \), the area is \( 4x^2 = 4 - y^2 \).

Then the volume is:
\[
\int_{-2}^{2} (4 - y^2) \, dy = (4y - y^3/3)_{-2}^{2} = 32/3.
\]

4. The average value of \( f(x) = \sin^2(x) \) on the interval \([0, 2\pi]\) is given by:
\[
\frac{1}{2\pi} \int_{0}^{2\pi} \sin^2(x) \, dx = \\
\frac{1}{2\pi} \int_{0}^{2\pi} \frac{1 - \cos(2x)}{2} \, dx = \frac{1}{4\pi} \left( x - \frac{1}{2}\sin(2x) \right)_{0}^{2\pi} = \frac{1}{4\pi} \left( 2\pi - \frac{1}{2}(\sin(4\pi) - \sin(0)) \right) = \\
\frac{1}{4\pi} (2\pi - 0) = \frac{1}{2}
\]
5(a) Letting $u = \sin^{-1}(x)$, then $du = dx/\sqrt{1-x^2}$, and the integral becomes $\int u \, du = u^2/2 + C = (\sin^{-1}(x))^2/2 + C$.

5(b) Use integration by parts with $u = \sin^{-1}(x)$, $dv = x \, dx/\sqrt{1-x^2}$, $du = dx/\sqrt{1-x^2}$, $v = -\sqrt{1-x^2}$. Then $\int x \sin^{-1}(x) \, dx = -\sqrt{1-x^2}\sin^{-1}(x) + \int dx = -\sqrt{1-x^2}\sin^{-1}(x) + x + C$.

5(c) Replace $\sin^2(x)$ by $1 - \cos^2(x)$ and use the substitution $u = \cos(x)$, $du = -\sin(x) \, dx$ to obtain:

$$\int (1 - \cos^2(x)) \cos^3(x) \sin(x) \, dx = \int (1 - u^2) u^4 \, du = -\frac{1}{5} u^5 + \frac{1}{7} u^7 + C = -\frac{1}{5} \cos^5(x) + \frac{1}{7} \cos^7(x) + C.$$

5(d) Since the equation $x^2 - 4x + 8 = 0$ has complex roots, we complete the square: $x^2 - 4x + 8 = (x - 2)^2 + 4$. Thus, we use the substitution $u = x - 2$, $du = dx$. Then, $\int_0^4 \frac{x}{x^2 - 4x + 8} \, dx = \int_{-2}^{2} \frac{2 \, du}{u^2 + 4} + 2 \int_{-2}^{2} \frac{du}{u^2 + 4} = \left[ \frac{1}{2} \ln(u^2 + 4) + \tan^{-1}(u/2) \right]_{-2}^{2} = \pi/2$.

5(e) Use the substitution $x = 2 \tan \theta$, then $dx = 2 \sec^2 \theta \, d\theta$, and $(4 + x^2)^{1/2} = 2 \sec \theta$. Thus,

$$\int \frac{dx}{(4 + x^2)^{3/2}} = \int \frac{2 \sec^2 \theta \, d\theta}{8 \sec^3 \theta} = \frac{1}{4} \int \frac{d\theta}{\sec \theta} = \frac{1}{4} \int \cos \theta \, d\theta = \frac{1}{4} \sin(\theta) + C = \frac{x}{4 \sqrt{4 + x^2}} + C.$$

5(f) Let $u = e^x + 1$, $du = e^x \, dx$, $e^x = u - 1$. Then, $\int e^{2x}/(1 + e^x)^{1/3} \, dx = \int e^x \cdot e^x/((1 + e^x)^{1/3}) \, dx = \int \frac{(u - 1) \, du}{u^{1/3}} = \int (u^{2/3} - u^{-1/3}) \, du = \frac{3}{5} u^{5/3} + \frac{3}{2} u^{2/3} + C = \frac{3}{5}(1 + e^x)^{5/3} + \frac{3}{2}(1 + e^x)^{2/3} + C$.

5(g) Here, $x^4 + 3x^3 + 2x^2 + x - 3 = (x^2 + 1) + 3x^3 + 2x^2 + x - 2$. Using partial fractions,

$$\frac{3x^3 + 2x^2 + x - 2}{(x-1)(x+1)(x^2+1)} = \frac{1}{x-1} + \frac{1}{x+1} + \frac{x}{x^2+1} + \frac{2}{x^2+1} \implies \frac{3x^3 + 2x^2 + x - 2}{x^4-1}.$$

Thus, $\int \frac{x^4 + 3x^3 + 2x^2 + x - 3}{x^4-1} \, dx = \int \left( \frac{1}{x-1} + \frac{1}{x+1} + \frac{x}{x^2+1} + \frac{2}{x^2+1} \right) \, dx = x + \ln|x-1| + \ln|x+1| + \frac{1}{2} \ln(x^2+1) + 2 \tan^{-1}x + C$.

6. In the problem, $f(x) = \cos(x^2)$, $f'(x) = -2x \sin(x^2)$, and $f''(x) = -2 \sin(x^2) - 4x^2 \cos(x^2)$. First we determine an upper bound $K$ for $f''(x)$ on the interval $[0, 1]$. On $[0, 1]$, $|x^2| \leq 1$, $|\sin(x^2)| \leq 1$, and $|\cos(x^2)| \leq 1$. Thus, $|f''(x)| = |\sin(x^2) + 4x^2 \cos(x^2)| \leq 2|\sin(x^2)| + 4|x^2||\cos(x^2)| \leq 2(1) + 4(1) = 6$. Thus we may take $K = 6$.

When the midpoint rule is used, an upper bound for the error is $K(b - a)^3/24n^2$. In this case, $K = 6$ and $b - a = 1$, and an upper bound for the error is $1/4n^2$. To ensure that the error is at most $1/10^6$, we choose $n$ sufficiently large so that $1/4n^2 \leq 1/10^6$, or $10^6 \leq 4n^2$, or $10^3 \leq 2n$, or $500 \leq n$.

7. (a), (c) converge; (b), (d) diverge.

7(a) $\int_1^\infty x^{-3/2} \, dx = -2x^{-1/2}\bigg|_1^\infty = 2$.

7(b) $\int_0^1 (1 - x)^{-3/2} \, dx = 2(1 - x)^{-1/2}\bigg|_0^1 = \infty$.

7(c) $\int_2^\infty \frac{dx}{x^2 - 1} = \int_2^\infty \frac{1}{2} \left( \frac{1}{x-1} - \frac{1}{x+1} \right) \, dx = \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \bigg|_2^\infty = 0 - \frac{1}{2} \ln(1/3) = \frac{1}{2} \ln 3$.
7(d) \[ \int_{2}^{\infty} \frac{x}{x^2 - 1} \, dx = \frac{1}{2} \ln |x^2 - 1| \bigg|_{2}^{\infty} = \infty. \]

8(a) The area is \[ \int_{0}^{\infty} e^{-x} \, dx = -e^{-x} \bigg|_{0}^{\infty} = 1. \]

8(b) Suppose that \( x = a \) divides \( R \) into two regions of the same area. Then, \[ \int_{0}^{a} e^{-x} \, dx = -e^{-x} \bigg|_{0}^{a} = -e^{-a} + 1 = 1/2, \]
or \( e^{-a} = 1/2 \), or \( \ln(e^{-a}) = \ln(1/2) \), or \( -a = -\ln 2 \), or \( a = \ln 2 \).