Math 581, Homework 4, due 3/21/06

Radoš Radoičić

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You are required to hand in 15 of the following problems.

Problems 1 – 26 from the book (electronic edition):

Chapter 5 – Problems: 6, 11, 12, 13, 15, 16, 18, 23, 28, 31, 32, 36, 37, 38, 39, 41, 43, 45.

Chapter 6 – Problems: 2, 3, 7, 10, 12, 13, 15, 21.

Problem 27: Let G be a graph in which no vertex lies on as many as $\binom{k}{2}$ odd cycles. Prove that G is k-colorable.

Problem 28: difficult We define a graph modeling perpendicular directions in space. The vertex set of G is the set of points on a sphere centered at the origin in $\mathbb{R}^3$ (this is an infinite graph). Make points p and q adjacent if the lines from the origin through p and through q are perpendicular. Determine $\chi(G)$.

Problem 29: difficult The Kneser graph $K(n,k)$ is the disjointness graph of the k-element subsets of $[n]$. That is, the vertex set consists of all k-element subsets of $[n]$, and two vertices are adjacent if they are disjoint k-sets. For example, the Petersen graph is $K(5,2)$. Prove that $\chi(K(n,k)) \le n - 2k + 2$ by covering the vertices with $n - 2k + 2$ independent sets. Prove that this is optimal when $n = 2k + 1$. (Note: Lovász ('78) proved Kneser's conjecture that $\chi(K(n,k)) = n - 2k + 2$ always holds.)