

Math 581, Homework 2, due 2/10/06

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February 2, 2006

You are required to hand in any 10 of the following problems.

Problems 1 – 16 from the book:

Chapter 2 – Problems: 5, 7, 8, 9, 10, 15, 16.

Chapter 3 – Problems: 6, 7, 9, 10, 11, 14, 17, 19, 20.

Problem 17: Mormons Let G be a bipartite graph with vertex classes $V_1 = \{x_1, \dots, x_m\}$ and $V_2 = \{y_1, \dots, y_n\}$. Then G contains a subgraph H such that $d_H(x_i) = d_i$ and $0 \leq d_H(y_j) \leq 1$ if and only if $|N(S)| \geq \sum_{x_i \in S} d_i$ for every $S \subset V_1$.

Problem 18: An $r \times s$ Latin rectangle based on $1, 2, \dots, n$ is an $r \times s$ matrix $A = (a_{ij})$ such that each entry is one of the integers $1, 2, \dots, n$, and each integer occurs in each row and column at most once. Prove that every $r \times n$ Latin rectangle can be extended to an $n \times n$ Latin square.

Problem 19: Let $m(n)$ be the maximal number of stable matchings in a set of n boys and n girls. Show that $m(n_1 + n_2) \geq m(n_1)m(n_2)$, and deduce that $m(n) \geq 2^{n/2}$.

Problem 20: Show that a graph with n vertices and m edges has an independent set of at least $2n/3 - m/3$ vertices. For which graphs is equality attained?