

Math 581, Homework 1, due 1/31/06

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You are required to hand in any 10 of the following problems.

Problems 1 – 13 from the book:

Chapter 1 – Problems: 3, 5, 6, 7, 8, 11, 14, 15, 17, 19, 20, 23, 24.

Problem 14: In the puzzle of the man and his wolf, goat, and cabbage, a man wishes to cross the river with his wolf, goat, and cabbage, but the small boat he has access to can take only one of his possessions besides himself. To complicate matters, for obvious reasons, the goat cannot be left in the company of the wolf or the cabbage, unless the man is also present. Advise the man how he should proceed.

Problem 15: Show that every graph G has a partition $V(G) = U \cup W$ such that $|E(U, W)| \geq \frac{1}{2}e(G)$.

Problem 16: Show that for every sequence of degrees $1 \leq d_1 \leq \dots \leq d_n \leq n - 1$, there are at most $(n - 2)!$ trees on $\{x_1, \dots, x_n\}$ with $d(x_i) = d_i$ for every i .

Problem 17: Prove that a regular bipartite graph of degree at least 2 does not have a bridge.

Problem 18: Show that a complete graph K^n has a decomposition into edge-disjoint paths of length 2 if and only if $n \equiv 0 \pmod{4}$ or $n \equiv 1 \pmod{4}$.

Problem 19: worth two problems Each of $n \geq 4$ professors know some item of gossip not known to others. They communicate by telephone and in each conversation they part with all the gossip they know. Show that $2n - 4$ calls are needed before each of them knows everything.

Problem 20: worth two problems Prove that every planar graph has a drawing in the plane in which every edge is a straight line segment.

Problem 21: Let \mathcal{T} be the set of spanning trees of a connected graph of order n . Let H be the graph with vertex set \mathcal{T} in which $T_1 \in \mathcal{T}$ and $T_2 \in \mathcal{T}$ are joined if T_1 has precisely one edge not in T_2 . Show that H is connected and has diameter at most $n - 2$.