SOLUTIONS—ASSIGNMENT 5

Chapter 3. Problems: p. 103 #1. Let E be "at least one 6", F be "different numbers". There are 36 outcomes, 6 in which the two dice are the same and hence 30 in which they are different, so $P(F) = 30/36 = 5/6$. There are 10 outcomes with different numbers in which one is a 6, so $P(EF) = 10/36 = 5/18$, and $P(E|F) = (5/18)/(5/6) = 1/3$. (This follows even more directly from the fact that the reduced sample space contains 30 equally likely outcomes, 10 of these having one '6'.)

p. 103 #7. The oldest boy (if there is one) of a royal family becomes king, so we know that the family has at least one boy. If G and B are the events that the family has a girl and has a boy, respectively, then

$$P(G|B) = P(GB)/P(B) = (1/2)\Big/(3/4) = 2/3.$$

Even more directly, notice that the reduced sample space is $\{(b,b),(b,g),(g,b)\}$.

p. 103 #8. The question just asks for the probability that the second child is a girl, which is 1/2. In terms of the formula for conditional probability, with G_1 the event that the first child is a girl, etc.,

$$P(G_1G_2|G_1) = P(G_1G_2)/P(G_1) = (1/4)\Big/(1/2) = 1/2.$$

p. 103 #9. Let A, B, and C be the event of choosing a white ball from the corresponding urn; $T = ABC^c \cup AB^cC \cup A^cBC$, the event that exactly two white balls are chosen. Then

$$\begin{aligned} P(A|T) &= \frac{P(AT)}{P(T)} = \frac{P(ABC^c) + P(AB^cC)}{P(T)} \\ &= \left[\frac{2}{6}\cdot\frac{8}{12}\cdot\frac{3}{4} + \frac{2}{6}\cdot\frac{4}{12}\cdot\frac{1}{4}\right]\Big/\left[\frac{2}{6}\cdot\frac{8}{12}\cdot\frac{3}{4} + \frac{2}{6}\cdot\frac{4}{12}\cdot\frac{1}{4} + \frac{4}{6}\cdot\frac{8}{12}\cdot\frac{1}{4}\right] \end{aligned}$$

p. 103 #10. The probability that the 1^{st}, 2^{nd} and 3^{rd} cards are spades is $\frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}$, while the probability that the 1^{st} is *not* a spade but the next two are is $\frac{39}{52}\cdot\frac{13}{51}\cdot\frac{12}{50}$. Hence

$$\begin{aligned} P(2^{\text{nd}} \text{ and } 3^{\text{rd}} \text{ are spades}) &= \frac{13\cdot12\cdot11 + 39\cdot13\cdot12}{52\cdot51\cdot50} = \frac{50\cdot13\cdot12}{52\cdot51\cdot50} \quad \text{and} \\ P(1^{\text{st}} \text{ is a spade}|2^{\text{nd}} \text{ and } 3^{\text{rd}} \text{ are spades}) &= \frac{P(1^{\text{st}},\ 2^{\text{nd}},\ 3^{\text{rd}} \text{ are spades})}{P(2^{\text{nd}} \text{ and } 3^{\text{rd}} \text{ are spades})} \\ &= \frac{13\cdot12\cdot11}{52\cdot51\cdot50}\Big/\frac{50\cdot13\cdot12}{52\cdot51\cdot50} = \frac{11}{50} = 0.22\ . \end{aligned}$$

Much easier: Use the reduced sample space for the first card given that the second and third cards are spades. There are $52 - 2 = 50$ possible (equally likely) cards, $13 - 2 = 11$ of which are spades. Hence the conditional probability is 11/50.

p. 105 #28. C: "color blind," M: "male," F: "female." Then if males and females are equally represented in the population, $P(M) = P(F) = 1/2$, so

$$P(M|C) = \frac{P(C|M)P(M)}{P(C|M)P(M) + P(C|F)P(F)} = \frac{(1/20)(1/2)}{(1/20)(1/2) + (1/400)(1/2)} = \frac{20}{21} = .952\ .$$

For the second part, take $P(M) = 2/3$, $P(F) = 1/3$ in this formula.

p. 105 #32. F: "first box," S: "second box," B: "black marble," W: "white marble." Then $P(B) = P(B|F)P(F) + P(B|S)P(S) = (1/2)(1/2) + (2/3)(1/2) = 7/12$; $P(F|W) = P(W|F)P(F)/P(W) = (1/2)(1/2)\Big/(5/12) = 3/5$.

p. 106 #37. Let M be the event that the present was hidden by mom, D by dad. Let U be the event that the present was hidden upstairs, L downstairs. Then

$$\begin{aligned} P(U) &= P(U|M)P(M) + P(U|D)P(D) \\ &= (.7)(.6) + (.5)(.4) \\ &= .62 \end{aligned}$$

and

$$\begin{aligned} P(D|L) &= \frac{P(L|D)P(D)}{P(L)} \\ &= \frac{(.5)(.4)}{1 - .62} = \frac{.20}{.38} = \frac{10}{19}. \end{aligned}$$

p. 106 #45. The events are: T: two-headed coin; F: fair coin; B: biased coin; H: heads. Then

$$\begin{aligned} P(T|H) &= \frac{P(H|T)P(T)}{P(H|T)P(T) + P(H|F)P(F) + P(H|B)P(B)} \\ &= \frac{1 \cdot (1/3)}{1 \cdot (1/3) + (1/2)(1/3) + (3/4)(1/3)}. \end{aligned}$$