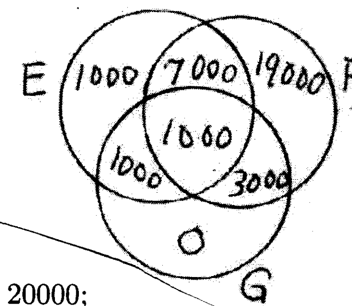
SOLUTIONS—ASSIGNMENT 3

Chapter 2. Problems: p 51 #8. a) .8 b) .3 c) 0

p. 51 #9. 74%

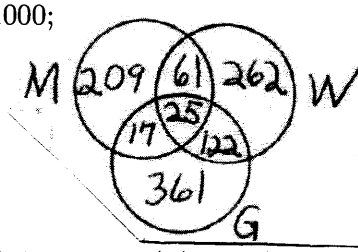
p. 51 #10. If R is the event of wearing a ring and N is the event of wearing a necklace, then $P(R^c \cap N^c) = 0.6$ so $P(R \cup N) = P((R^c \cap N^c)^c) = 1 - 0.6 = 0.4$. Since $P(R \cup N) = P(R) + P(N) - P(R \cap N)$, we have $0.4 = 0.2 + 0.3 - P(R \cap N)$ which implies $P(R \cap N) = 0.5 - 0.4 = 0.1$. Hence (a) $P(R \cup N) = 40\%$ wear one decoration or the other, while (b) $P(R \cap N) = 10\%$ wear both.

p. 51 #13. One approach is systematically to work out the number of people in each piece of a Venn diagram, then answer the questions. For example, let E, F, and G be the sets of people who read paper I, II, and III, respectively. We are given that $|EFG| = 1000$ and $|EF| = 8000$, so $|EFG^c| = 7000$. Similarly, $|EF^cG| = 1000$, and since $|E| = 10000$ we have that $|EF^cG^c| = 10000 - 7000 - 1000 - 1000 = 1000$. We fill in the entire Venn diagram in this manner.



Now (a) $|EF^cG^c| + |E^cFG^c| + |E^cF^cG| = 1000 + 19000 + 0 = 20000$;
(b) $|EFG^c| + |EF^cG| + |E^cFG| + |EFG| = 7000 + 1000 + 3000 + 1000 = 12000$;
(c) $|EFG| + |EFG^c| + |E^cFG| = 1000 + 7000 + 3000 = 11000$;
(d) $100,000 - 32,000 = 68,000$
(e) $|EFG^c| + |E^cFG| = 7000 + 3000 = 10000$.

p. 51 #14. Working out the Venn diagram as in the problem above shows that a total of 1057 people must have participated in the study.



Theoretical exercises: p. 55 #11. By Propositions 4.2 and 4.3, $1 = P(S) \geq P(E \cup F) = P(E) + P(F) - P(EF)$, and taking the term $P(EF)$ to the left side and the 1 to the right side yields Bonferroni's inequality. Applying this inequality with $P(E) = .8$ and $P(F) = .9$ gives $P(EF) \geq .9 + .8 - 1 = .7$.

p. 55 #12. The event that exactly one of the events E and F occurs is the event that either E occurs and F doesn't or that F occurs and E doesn't, i.e., that either EF^c or E^cF occurs; this is just the event $EF^c \cup E^cF$. Since $P(E) = P(EF) + P(EF^c)$ and $P(F) = P(EF) + P(E^cF)$, we have

$$\begin{aligned} P(EF^c \cup E^cF) = P(EF^c) + P(E^cF) &= P(E) - P(EF) + P(F) - P(EF) \\ &= P(E) + P(F) - 2P(EF). \end{aligned}$$

Self-Test: p. 57 #14. The text provides a nice proof. Another proof, for the case of a finite union rather than a general infinite union, is as follows:
First we note that for any events E and F, $P(EF) \geq 0$ and therefore from Proposition 4.3, $P(E \cup F) = P(E) + P(F) - P(EF) \leq P(E) + P(F)$. Now we can prove Boole's inequality by induction on n. For $n = 1$, Boole's inequality is just $P(A) \leq P(A)$, which is certainly true. Then if we assume that it holds for n, we can see that it holds for $n + 1$:

$$\begin{aligned} P\left(\bigcup_{i=1}^{n+1} A_i\right) &= P\left(\left(\bigcup_{i=1}^{n} A_i\right) \cup A_{n+1}\right) \leq P\left(\bigcup_{i=1}^{n} A_i\right) + P(A_{n+1}) \\ &\leq \sum_{i=1}^{n} P(A_i) + P(A_{n+1}) = \sum_{i=1}^{n+1} P(A_i). \end{aligned}$$