SOLUTIONS—ASSIGNMENT 2

Chapter 2.

Problems: p. 50 #1. $S = \{RR, RG, RB, GR, GG, GB, BR, BG, BB\}$

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p. 50 #2. A typical point in the sample space might look like 5 1 3 5 5 2 6; this means that we rolled the die seven times, getting our 6 on the last roll. Of course, in theory we might never get a 6. Thus the sample space consists of all finite sequences of integers of the form $k_1k_2k_3\ldots k_m6$, where $m \geq 0$, together with all infinite sequences $k_1k_2k_3\ldots$; here each integer k_i is 1, 2, 3, 4, or 5. Note that for $m = 0$ we have the outcome 6, meaning that we rolled a 6 on our first try. The event E_n consists of all sequences of total length n, i.e., those finite sequences above with $m = n - 1$. $\bigcup_1^\infty E_n$ consists of all finite sequences, so that $\left(\bigcup_1^\infty E_n\right)^c$ consists of all infinite sequences.

p. 50 #3. EF: roll a 1 and an even number (since the sum must be odd); $E \cup F$: roll a 1 or an odd sum (or both); FG: roll a 1 and a 4; EF^c: roll an odd sum with neither die showing 1; EFG: the same as FG, since $G \subset E$.

p. 50 #4. $A = \{1, 0001, 0000001, \ldots, \underbrace{00\ldots0}_{3n}1, \ldots\}$

$(A \cup B)^c = \{000\ldots, 001, 000001, \ldots, \underbrace{00\ldots0}_{3n+2}1, \ldots\}$

Theoretical exercises:

p. 55 #1:

$$E \cap F \subseteq E: \quad \text{if } x \in E \cap F \text{ then of course } x \in E.$$

$$E \subseteq E \cup F: \quad \text{if } x \in E \text{ then of course } x \in E \cup F.$$

p. 55 #2: Suppose $E \subseteq F$. If $x \in F^c$, then if x were in E we would have $x \in F$ contradicting $x \in F^c$; thus we have $x \in E^c$.

p. 55 #3: $FE \cup FE^c = F(E \cup E^c) = FS = F$. Using this, we have $E \cup F = (E \cup FE) \cup FE^c = E \cup FE^c$.

p. 55 #4: $x \in \left(\bigcup_{i=1}^{\infty} E_i\right) \cap F$ if and only if $x \in F$ and for some index i, $x \in E_i$. But this is the same as: For some index i, $x \in E_i \cap F$. The set of all such x is exactly $\bigcup_{i=1}^{\infty}(E_i \cap F)$.

p. 55 #6. (a) EF^cG^c (b) EGF^c (c) $E \cup F \cup G$ (d) $EF \cup EG \cup FG$ (e) EFG (f) $E^cF^cG^c$ (g) $EF^cG^c \cup E^cFG^c \cup E^cF^cG \cup E^cF^cG^c$ (h) $(EFG)^c$ (i) $EFG^c \cup EF^cG \cup E^cFG$ (j) S