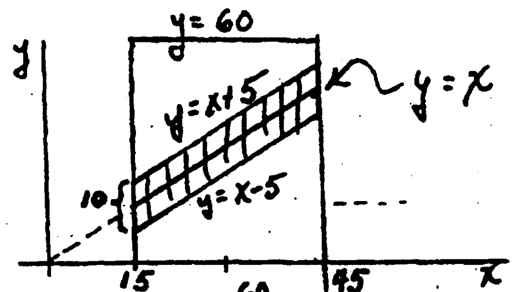
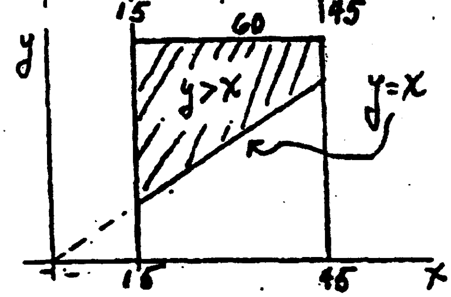
SOLUTIONS—ASSIGNMENT 16

Chapter 6. Problems: p. 292 #13. If X and Y are the arrival times of the man and woman, respectively, in minutes after noon, then (X, Y) is uniformly distributed on the rectangle $15 \leq x \leq 45$, $0 \leq y \leq 60$. We can thus determine probabilities geometrically, as ratios of areas.

(a) The set in which the early arriver must wait at most 5 minutes, $\{|x - y| \leq 5\}$, is a parallelogram of altitude 30 and base 10, so $P\{|X - Y| \leq 5\} = (10 \times 30)/(30 \times 60) = 1/6$.



(b) It seems clear from the symmetry between the man and the woman that we should have $P\{X < Y\} = 1/2$, and this is easily verified by computing the area of the corresponding trapezoid.



p. 292 #15. (b) The joint density is $f(x, y) = 1/4$ if $-1 \leq x, y \leq 1$, $f(x, y) = 0$ otherwise. Thus

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy = \begin{cases} \frac{1}{4}\int_{-1}^{1} dy = 1/2, & \text{if } -1 \leq x \leq 1, \\ 0, & \text{otherwise.} \end{cases}$$

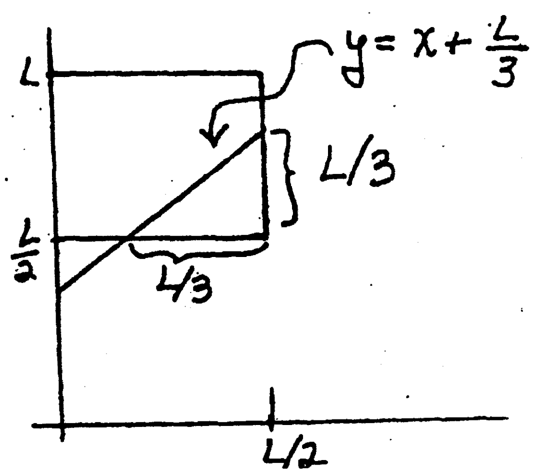
Clearly X is uniform on $(-1, 1)$. Similarly, Y is uniform on $(-1, 1)$. Moreover, X and Y are independent because $f(x, y) = f_X(x) f_Y(y)$.

(c) Let A be the region inside the circle of radius 1, centered at the origin. Then

$$P\{X^2 + Y^2 \leq 1\} = P\{(X, Y) \in A\} = \frac{|A|}{4} = \pi/4.$$

Ninth edition #18. X and Y are uniformly distributed on the square $0 \leq X \leq L/2$, $L/2 \leq Y \leq L$. The set where $Y - X > L/3$ is the complement of a triangle with base and height $L/3$, so

$$P\{Y > X + L/3\} = 1 - \frac{(L/3)^2(1/2)}{(L/2)^2} = \frac{7}{9}.$$



p. 293#20.(a) Integrating over y yields $f_X(x) = xe^{-x}$ for $x \geq 0$, and similarly $f_Y(y) = e^{-y}$ for $y \geq 0$; f_X and f_Y are zero if their arguments are negative. Since $f(x,y) = f_X(x)f_Y(y)$, X and Y are independent.

(b) Here $f_X(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{otherwise,} \end{cases}$; $f_Y(y) = \begin{cases} 2y, & 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$. X and Y are not independent.