Quiz #9 Solutions
Math 250, Section B1
June 30, 2009

1. If $\mathbf{w}$ is the orthogonal projection of $\mathbf{u}$ onto the line spanned by $\mathbf{v}$, what can you say about $(\mathbf{u}-\mathbf{w}) \bullet 3\mathbf{v}$?
   Solution: It must be zero, since $\mathbf{u}-\mathbf{w}$ will be orthogonal to every vector on the line spanned by $\mathbf{v}$.

2. Suppose $\|\mathbf{u}\| = 2$, $\|\mathbf{v}\| = 3$, and $\mathbf{u} \bullet \mathbf{v} = 4$. Determine $\|\mathbf{u} + 2\mathbf{v}\|$.
   (Hint: First find $\|\mathbf{u} + 2\mathbf{v}\|^2$.)
   Solution: Since

$$\begin{aligned} \|\mathbf{u}+2\mathbf{v}\|^2 &= (\mathbf{u}+\mathbf{2v}) \bullet (\mathbf{u}+\mathbf{2v}) \\ &= \mathbf{u} \bullet \mathbf{u} + \mathbf{u} \bullet (2\mathbf{v}) + (2\mathbf{v}) \bullet \mathbf{u} + (2\mathbf{v}) \bullet (2\mathbf{v}) \\ &= \|\mathbf{u}\|^2 + 4\mathbf{u} \bullet \mathbf{v} + 4\|\mathbf{v}\|^2 \\ &= 2^2 + 4 \cdot 4 + 4 \cdot 3^2 \\ &= 56, \end{aligned}$$

   we conclude that $\|\mathbf{u} + 2\mathbf{v}\| = \sqrt{56} = 2\sqrt{14}$.

3. Determine whether each of the following statements is `TRUE` or `FALSE`. Justify each `FALSE` answer. (You don't have to justify `TRUE` answers.)

   (a) If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal vectors in $\mathcal{R}^n$, then $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$.
   Solution: `FALSE`. For example, if $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal but $\|\mathbf{u} + \mathbf{v}\| \neq \|\mathbf{u}\| + \|\mathbf{v}\|$.

   (b) If $\|\mathbf{v}\| = 0$, then it must be the case that $\mathbf{v} = \mathbf{0}$.
   Solution: `TRUE`