Consider the curve \( \mathbf{r}(t) = (t, t^2, \frac{2}{3} t^3) \), a modified "twisted cubic."

- Find the length of the curve between \( t = 0 \) and \( t = 2 \).
- Compute the curvature of \( \mathbf{r} \) at \( t = 1 \).

\[
\mathbf{r}'(t) = \langle 1, 2t, 2t^2 \rangle
\]

\[
\|\mathbf{r}'(t)\| = \sqrt{1 + 4t^2 + 4t^4} = \sqrt{(2t^2 + 1)^2} = 2t^2 + 1.
\]

\[
\int_0^2 \|\mathbf{r}'(t)\| \, dt = \int_0^2 (2t^2 + 1) \, dt = \frac{2}{3} t^3 + t \bigg|_0^2 = \frac{16}{3} + 2 = \frac{22}{3}.
\]

\[
\mathbf{r}'(t) = \langle 1, 2t, 2t^2 \rangle
\]

\[
\mathbf{r}''(t) = \langle 0, 2, 4t \rangle.
\]

\[
\mathbf{r}'(t) \times \mathbf{r}''(t) = \langle 4t^2, -4t, 2 \rangle.
\]

\[
\|\mathbf{r}'(t) \times \mathbf{r}''(t)\| = \sqrt{16t^4 + 16t^2 + 4} = 2 \sqrt{4t^4 + 4t^2 + 1} = 2(2t^2 + 1).
\]

\[
S_0 \int_0^2 \mathbf{r}'(t) \times \mathbf{r}''(t) \, dt = \frac{2}{3} \sqrt{4t^4 + 4t^2 + 1} = \frac{2}{3} \sqrt{(2t^2 + 1)^2}.
\]

\[
\kappa(t) = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3} = \frac{2(2t^2 + 1)}{(2t^2 + 1)^3} = \frac{2}{(2t^2 + 1)^{\frac{2}{3}}}.
\]

\[
\kappa(1) = \frac{2}{(2 + 1)^2} = \frac{2}{9}.
\]