Please see the homework guidelines on the course page.
If something seems wrong, please ask before wasting a lot of time on it.
All problem parts have equal weight.

1. Find (with justification) the number of sequences \( \Pi_1, \ldots, \Pi_n \) satisfying
   (i) for each \( i \), \( \Pi_i \) is an unordered partition of \( [n] \) into \( i \) nonempty blocks (so \( \Pi_1 = \{[n]\} \) and \( \Pi_n \) is the partition into singletons), and
   (ii) for \( i = 2, \ldots, n \), \( \Pi_i \) is gotten from \( \Pi_{i-1} \) by splitting some block (necessarily of size at least 2) into two nonempty blocks.

2. Find a closed form for \( \sum_{i=0}^{k} (-1)^i \binom{n}{i} \binom{n}{k-i} \).

3. Assume \( n \sim k^2 \). For fixed \( X \in \mathcal{K} := \binom{[n]}{k} \) and \( Y \) chosen uniformly from \( \mathcal{K} \), give (and justify) asymptotics for \( \Pr(Y \cap X = \emptyset) \).

4. For fixed \( k \) and \( n \to \infty \), give an asymptotic expression for \( |s(n,k)| \) (and justify, of course).

5.(a) Give a simple (one sentence?) combinatorial explanation of the identity

   \[ \sum_{k=0}^{n} \binom{k}{s} = \binom{n+1}{s+1}. \]

   (b) Use (a) to express \( \sum_{k=0}^{n} k^4 \) as a linear combination of a small number (not depending on \( n \)) of binomial coefficients.

6. Show (via bijection) that the Bell number \( B_n \) is equal to the number of permutations \( a_1, \ldots, a_n \) of \( [n] \) with the property that for no \( 1 \leq i < j \leq n - 1 \) do we have \( a_i < a_j < a_{j+1} \).

   [Here I’m mainly interested in the correspondence: you don’t need to justify at great length.]