Asymptotic notation

These are used for $f, g$ functions of some parameter, e.g. $n$ or $x$, which, as here, is often suppressed in the notation. The limiting statements are meant as the parameter approaches some limit (most often $n \to \infty$); the others are for the parameter in some range (with the limit or range for the parameter either specified or clear from the context).

\[
\begin{align*}
f \sim g: & \quad f/g \to 1 \\
f = O(g): & \quad |f|/|g| \text{ is bounded above} \\
f = o(g): & \quad f/g \to 0 \\
f = \Omega(g): & \quad g = O(f) \quad \text{(equiv: } |f|/|g| \text{ is bounded below by a positive constant)} \\
f = \omega(g): & \quad |f|/|g| \to \infty \quad \text{(equiv: } g = o(f)) \\
f = \Theta(g): & \quad f = O(g) \text{ and } g = O(f) \quad \text{(equiv: } |f|/|g| \text{ lies between two positive constants)} \\
f \preccurlyeq g: & \quad \limsup f/g \leq 1 \quad \text{(not sure we’ll see this one)}
\end{align*}
\]

We can then, for example, write simply $O(g)$ to mean any (perhaps unspecified) function whose absolute value is known to be bounded above by $Cg$ for some fixed $C$. Big and little “Oh” are often used for error terms, for example

\[
e^x = 1 + x + O(x^2) \quad \text{as } x \to 0,
\]

in which case the functions $O(\cdot)$, $o(\cdot)$ will often be negative. In most (?) of our other uses $f$ and $g$ will be positive.