JAKE BARON'S ORAL QUAL SYLLABUS

COMMITTEE: JEFF KAHN (CHAIR), SIMON THOMAS, JÓZSEF BECK, DORON ZEILBERGER

1. Combinatorics

**Enumeration**: bijections, generating functions, binomial and multinomial coefficients, recurrence relations, inclusion-exclusion, Stirling’s formula

**Hypergraphs**: Sperner, LYM inequality, Erdős-Ko-Rado, Kruskal-Katona, Fisher’s Inequality (and generalized Fisher), Ray-Chaudhuri-Wilson, Frankl-Wilson, Harper, Baranyai

**Posets and Lattices**: Dilworth, linear extensions of posets, \(\frac{1}{4}\)-conjecture, distributive and geometric lattices, Birkhoff representation theorem, Möbius inversion, Weisner, Dowling-Wilson

**Correlation Inequalities**: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre, Ahlswede-Daykin, Shepp XYZ

**Discrepancy**: Beck-Fiala, Roth’s \(\frac{1}{4}\)-theorem on arithmetic progressions

**Ramsey Theory**: Ramsey, infinite Ramsey, König tree lemma, probabilistic lower bounds, van der Waerden, statement of Szemerédi

2. Probabilistic Methods

**Basics**: linearity of \(E\), \(\cup\)-bound and Bonferroni inequalities, Chebychev’s inequality, Chernoff bounds, alteration methods

**Second Moment Method**: application to threshold function for containing a fixed subgraph

**Local Lemma**: symmetric and general versions, applications to hypergraph discrepancy, Ramsey lower bounds, Latin transversals

**Poisson Paradigm**: Janson’s inequality and application to number of triangles in \(G_{n,p}\), Brun’s sieve and application to number of isolated vertices in \(G_{n,p}\)

**Martingales**: vertex and edge exposures, Azuma’s inequality and application to chromatic number

**Random Graphs**: \(G_{n,p}\) vs. \(G_{n,M}\), monotone properties, existense of threshold functions, Bollobás-Thomason, probabilistic refutation of Hajós conjecture

**Entropy**: basic properties, Shearer’s lemma, application to Minc Conjecture

**Dependent Random Choice**: Balog-Szemerédi-Gowers

3. Graph Theory

**Matchings**: König, Hall, Tutte, stable matchings, matching polytopes

**Connectivity**: Kruskal’s algorithm for minimum weight spanning tree, Menger, max-flow-min-cut, structure of 2-connected graphs

**Planarity**: Euler’s formula, Kuratowski, Wagner

**Coloring**: 5-color theorem, Brooks, Vizing, Thomasson’s 5-list-coloring of planar graphs, perfect graphs, Lovász’s proof of weak perfect graph theorem, Galvin’s proof of Dinitz conjecture

**Extremal**: Turán, statement of regularity lemma, Erdős-Stone, Chvátal-Rödl-Szemerédi-Trotter

4. Foundations

**Recursion Theory**: recursive functions, Ackerman function, turing machines, Church-Turing thesis (statement), computable and recursively enumerable sets, turing degrees, jump operator, halting problem

**Model Theory**: Gödel completeness, Henkin theories, compactness, ultrafilters, ultraproducts, Loś, types, omitting types, Löwenheim-Skolem

**Basic Set Theory**: ordinals, cardinals, cardinal arithmetic, König’s lemma, equivalent forms of Choice, \(\emptyset\), MA, Aronszajn trees, Suslin trees, Root system lemma

**Forcing**: forcing theorems (statements), types of ultrafilters on \(\omega\) (Ramsey, selective, weakly selective)

**Descriptive Set Theory**: Polish and standard Borel spaces, Borel isomorphism theorem, Borel and projective hierarchies