“Paradoxes” in Game Theory

Or, Why I’m Not an Economist

Jake Baron
What is Game Theory?
(first attempt)
What is Game Theory?
(first attempt)

• Analysis of “strategic situations”
What is Game Theory? 
(philosophically)
What is Game Theory? (philosophically)

• Three views:
  – Descriptive
  – Proscriptive
  – Pure mathematics
What is Game Theory?
(mathematically)
What is Game Theory? (mathematically)

• Optimization over a Cartesian product
What is a game (mathematically)?
[i.e. it’s time for some real math now]
What is a game (mathematically)?
[i.e. it’s time for some real math now]

• Record all relevant information in a tuple

\[(P, (S_i)_{i \in P}, (u_i)_{i \in P})\]

$P$ is a nonempty set of **players**

$S_i$ is a nonempty set of **strategies** for player $i$

$u_i: \prod_{j \in P} S_j \to \mathbb{R}$ is a **utility function** for player $i$
Traveler’s Dilemma
(Kaushik Basu, 1994)

What should I say my urn is worth? ($2 to $100, integer number of $)
<table>
<thead>
<tr>
<th>Alice</th>
<th>$100</th>
<th>$99</th>
<th>$98</th>
<th>$97</th>
<th>...</th>
<th>$3</th>
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<td>...</td>
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</table>
Here’s a simpler game

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Center</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Alice</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>2,3</td>
<td>4,8</td>
<td>5,2</td>
</tr>
<tr>
<td>Medium</td>
<td>0,5</td>
<td>6,1</td>
<td>3,0</td>
</tr>
<tr>
<td>Low</td>
<td>1,4</td>
<td>0,2</td>
<td>5,5</td>
</tr>
<tr>
<td><strong>Bob</strong></td>
<td></td>
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Here’s a simpler game

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<td>6, 1</td>
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<tr>
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<td>0, 2</td>
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Let’s play!
Here’s a simpler game

- For Alice, Low is *dominated* by High
Here’s a simpler game

- For Alice, Low is *dominated* by High
Here’s a simpler game

- For Alice, Low is *dominated* by High
- Now for Bob, Right is dominated by Center
Here’s a simpler game

- For Alice, Low is *dominated* by High
- Now for Bob, Right is dominated by Center
Here’s a simpler game

- For Alice, Low is \textit{dominated} by High
- Now for Bob, Right is dominated by Center
- So the only \textit{rationalizable} outcomes are in \{High, Medium\} x \{Left, Center\}
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Dominance and Traveler’s Dilemma

• Only “rationalizable” outcome is ($2, $2)!
Dominance and Traveler’s Dilemma

• Only “rationalizable” outcome is ($2, $2)!

• wtf?

• Surely we have a better solution concept than this...
Chain Store Paradox
(Reinhard Selten, 1978)

20 neighborhood burger joints, one by one

- Neighborhood joints: enter market or not?
- If so, McDonalds cooperative or aggressive?
Backwards Induction

- At stage 20, **McD’s** should always **Coop** if **NJ20** is **In**
Backwards Induction

- At stage 20, McD’s should always Cooperate if NJ20 is In
- Thus NJ20 should get In
Backwards Induction

• At stage 20, **McD’s** should always **Coop** if **NJ20** is **In**
• Thus **NJ20** should get **In**
• So stage 20 is determined. Thus at stage 19... etc.
Backwards Induction

- At stage 20, McD’s should always Cooperate if NJ20 is In
- Thus NJ20 should get In
- So stage 20 is determined. Thus at stage 19... etc.
- This yields 40 for McD’s and 2 for each nbhd joint
But Wait!

• Can McDonald’s do better by acting “irrationally?”
But Wait!

• Can McDonald’s do better by acting “irrationally?”
• What if McDonald’s is aggressive in the first round?
Selten’s Explanation

• Three “levels” of decision-making:
  – “routine” (past experience)
  – “imagination” (how will my decision affect future?)
  – “reasoning” (“rational” analysis, i.e. game theory)
Another possible direction: $n$th order rationality

- Keynesian Beauty Contest

- Guess 2/3 of the average guess
Punch line?

And why am I not an economist?