Workshop 5 - Problems
Mon. June 29, 2009

Problem 1. Given that $y_1(t) = e^{-5t}$ is a solution to the differential equation $dy/dt + a(t)y = 0$, what is the solution with initial condition $y(1) = 1$?

Problem 2. When Euler's method is performed on the initial value problem $dy/dt = f(t, y)$, $y(0) = 3$ over the interval $-2 \le t \le 0$, the table of values obtained includes $(t_7, y_7) = (-1.75, 4.692)$. What step size Δt is being used?

Problem 3. Let $y(t)$ be the solution to the initial value problem

$$\frac{dy}{dt} = e^y \cos \pi t, \;\; y(-5) = 0.$$

What is the equation of the line tangent to the graph of $y(t)$ at the point $t = -5$?

Problem 4. Determine all the locations where the slope marks for the equation

$$\frac{dy}{dt} = -t^3 + y^3 + t^2 y - t y^2 + t - y$$

are horizontal.

Problem 5. Find one solution to the differential equation

$$\frac{dS}{dt} = 3 - \frac{4S}{7 - t}.$$

Problem 6. Suppose that y is an equilibrium point of the autonomous equation $\frac{dy}{dt} = f_\mu(y)$ if and only if $2y^3 + 3y^2 - 12y = \mu - 1$. At which values of μ does this family experience a bifurcation?

Problem 7. Find an expresion $f(t, y)$ such that the function

$$y(t) = t + \int_0^t e^{-u^2} du$$

is a solution of $dy/dt = f(t, y)$.