**Workshop 4 - Problems**

Mon. June 22, 2009

**Problem 1.** For which values of $\mu$ will the system

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2 & 3 \\ \mu & -2 \end{pmatrix} \mathbf{Y}$$

be a center?

**Problem 2.** Find **one** expression $f(x, y)$ such that the system

$$\begin{aligned} \frac{dx}{dt} &= f(x, y) \\ \frac{dy}{dt} &= 3e^{\sin(x+y)} \end{aligned}$$

has solution curves which are portions of parabolas.

**Problem 3.** The coefficient matrix of a linear system $d\mathbf{Y}/dt = \mathbf{A}\mathbf{Y}$ satisfies

$$A \begin{pmatrix} 5 \\ -7 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \text{ and } A \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

What is the value of

$$A \begin{pmatrix} 2 \\ -3 \end{pmatrix}?$$

**Problem 4.** For some linear system $d\mathbf{Y}/dt = \mathbf{A}\mathbf{Y}$, the vector field mark at the point $(2, -3)$ is the arrow $(-1, \frac{3}{2})$. Find **one** solution to this system.

**Problem 5.** Given that

$$e^{At} = \begin{pmatrix} e^{2t} & e^{5t} - e^{2t} \\ 0 & e^{5t} \end{pmatrix},$$

what is $e^{Bt}$ where $B = SAS^{-1}$ and

$$S = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}?$$

**Problem 6.** When Euler's method is used on the initial-value problem

$$\frac{d\mathbf{Y}}{dt} = \mathbf{F}(t, \mathbf{Y}), \;\; \mathbf{Y}\left(\frac{5}{3}\right) = \begin{pmatrix} -1 \\ 4 \end{pmatrix},$$

the table of values includes

$$\mathbf{Y}_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

and

$$\mathbf{Y}(3) \approx \mathbf{Y}_2 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}.$$

What is the value of

$$\mathbf{F}\left(\frac{5}{3}, \begin{pmatrix} -1 \\ 4 \end{pmatrix}\right)?$$

**Problem 7.** Suppose that the autonomous system $d\mathbf{Y}/dt = \mathbf{F}(\mathbf{Y})$ has the pair of functions $x_1(t) = t^3 + 30\pi^3$, $y_1(t) = \cos(t) - 5$ as a solution. Let $x_2(t)$, $y_2(t)$ be the solution with initial condition $x_2(0) = 3\pi^3$, $y_2(0) = -6$. Is it possible to find **explicit** formulas for $x_2(t)$ and $y_2(t)$ given this information? If so, find them.