Workshop 3 - Problems

Fri. June 19, 2009

Problem 1. Convert the third-order differential equation

$$\frac{d^3y}{dt^3} - t\frac{d^2y}{dt^2} + 3\cos t\frac{dy}{dt} - e^t y + 1 = 0$$

to a system of first-order differential equations.

Problem 2. Find all solutions to

$$\begin{aligned}\frac{dx}{dt} &= (x+3)(y-2)\\ \frac{dy}{dt} &= (x-5)y\end{aligned}$$

for which there exists t_0 such that $x(t_0) = 5$ and $y(t_0) = 2$.

Problem 3. Write a system that models the amounts $a(t)$ and $b(t)$ of radioactive elements A and B if element A decays into element B, and element B decays into a stable isotope of the element C.

Problem 4. Are there numbers a and b such that the pair of functions $x(t) = a\sin t + e^{2t}$, $y(t) = \cos t + be^{2t}$ is a solution to the following system?

$$\begin{aligned}\frac{dx}{dt} &= 3y + (a-3)\cos t\\ \frac{dy}{dt} &= x + (2b-1)e^{2t}\end{aligned}$$

If so, find such numbers.

Problem 5. The exponential $e^{\Lambda t}$ of an upper-triangular matrix Λ is

$$\begin{pmatrix} e^{-t} & \frac{1}{2}e^{3t} - \frac{1}{2}e^{-t} \\ 0 & e^{3t} \end{pmatrix}.$$

What are the eigenvalues of Λ?

Problem 6. Some system of differential equations has as particular solutions the pair of functions $x_1(t) = 3 - 3\sin t$, $y_1(t) = 2 + 3\cos t$ and the pair of functions $x_2(t) = -t$, $y_2(t) = -1$. Find all the points of intersection between the solution curves corresponding to these two particular solutions of the system.

Problem 7. Given that $\mathbf{F}(x, y) = (-2, 2)$ for all (x, y) on the line $y = -x - 1$, find **one** solution to the autonomous system of differential equations $\frac{d\mathbf{Y}}{dt} = \mathbf{F}(\mathbf{Y})$.