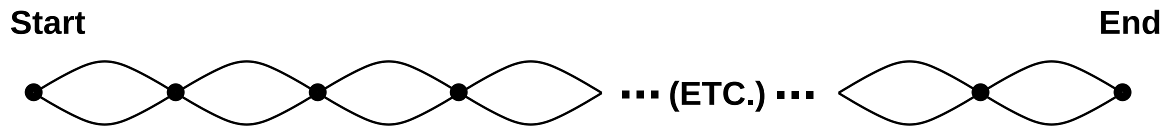
For Thursday, July 20 **YSP problems** Summer 2006

You should know about analysis of the **transmission** and **broadcasting** networks done in class.

1. Suppose your network looks like this:



This is a network with doubled links and n steps.

That is, in order to increase the likelihood of success, we add redundancy. Suppose that the probability of any one link breaking is $q = 1 - p$, so p is the probability that a link transmits. Also suppose that the links break independently.

a) Find W_n, a formula for the probability that a message can be passed from the START to the END.

b) Compute W_{50} when $p = .999$ and compare this to $(.999)^{50} \approx .95121$, the probability that a single-linked network with 50 "steps" is unbroken.

2. Suppose your network looks like this:

This whole network is doubled and has n steps.

That is, in order to increase the likelihood of success, we add redundancy, here in the form of two long connections. Suppose that the probability of any one link breaking is $q = 1 - p$, so p is the probability that a link transmits. Also suppose that the links break independently.

a) Find V_n, a formula for the probability that a message can be passed from the START to the END.

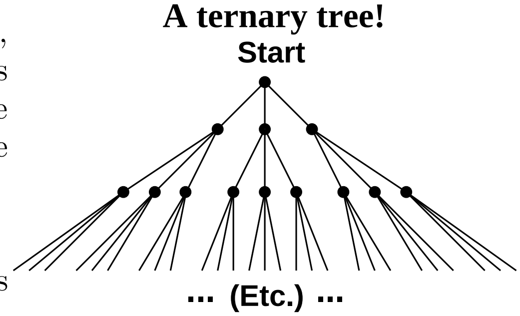
b) Compute V_{50} when $p = .999$ and compare this to $(.999)^{50} \approx .95121$, the probability that a single-linked network with 50 "steps" is unbroken.

3. Please keep the same assumptions on the links (p, q, independence, etc.), and consider a *ternary tree* as shown. Suppose K_n is the probability that a message can be passed from the *root* (called START in the diagram) to at least one leaf at the n^{th} level.

a) Verify that $K_{n+1} = 1 - (1 - pK_n)^3$.

b) Knowing that $K_0 = 1$, compute the polynomials K_1, K_2, and K_3.

A ternary tree!



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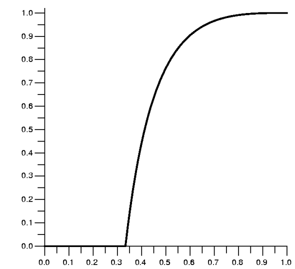
4. Assume that $K_{n+1} = 1 - (1 - pK_n)^3$ for the ternary tree. Now assume that as n gets large, $K_n \to \mathcal{K}$ where $\mathcal{K}$ depends on the probability, p. Find a formula for $\mathcal{K}$ as a function of p. Graph this function. Remember that for $p < \frac{1}{3}$ the limiting probability is 0 because there is not enough "fanout" to prevent K_n from dropping to 0. What is $\mathcal{K}$ when $p = .7$? **Hint** When p=.7, $\mathcal{K}\approx$.966, I think.

More hint



5. What happens if the tree branches 4 times at each node? What's the equation look like, what happens if you try to "solve" for the limiting probability, what ...?

6. Suppose the branching gets larger and larger. What happens to the curve of the limiting probability? (Just *guess* but think about your guess a bit!)

Useful background quotes

John von Neumann (1903-1957) was a mathematician who was raised in Hungary and spent most of his career in the United States. He worked in many areas of pure and applied mathematics. His ideas were influential in quantum mechanics, the development of nuclear weapons, game theory, and the theory and construction of digital computers. Here are some characteristic quotes:

> *Any one who considers arithmetical methods of producing random digits is, of course, in a state of sin.*
>
> This is in connection with the use of pseudorandom number generators. Von Neumann was one of the early users of such programs, but cautioned that they had to be carefully investigated.
>
> *It would appear that we have reached the limits of what it is possible to achieve with computer technology, although one should be careful with such statements, as they tend to sound pretty silly in 5 years.*
>
> *In mathematics you don't understand things. You just get used to them.*
>
> *You insist that there is something that a machine can't do. If you will tell me precisely what it is that a machine cannot do, then I can always make a machine which will do just that.*

Just one more nice quote, this one from Richard Hamming (1915–1998), who was a very important applied mathematician of the twentieth century. He did important work on (again, [sigh!]) atomic bombs, but his name is attached to fundamental results in error-correcting codes, numerical analysis, signal processing, information theory, etc. Here is an *extremely* useful aphorism of Hamming. I have sometimes forgotten it in my own work, and I am usually sorry as a result.

> *The purpose of computing is insight, not numbers.*