TV Review Problems

(1) Find the domain and range of the following functions:
a) $f(x) = \sqrt{x+3}$ b) $g(x) = \frac{4}{x^4+1}$ c) $h(x) = \sin(x)$

(2) True or false:
a) $2^a 3^b = 6^{a+b}$ b) $2^a 2^b = 2^{a+b}$ c) $\ln(a+b) = (\ln a)(\ln b)$
d) $e^{\ln a - \ln b} = a/b$

(3) Evaluate the limits if possible or state that it does not exist.
a) $\lim_{x\to 2} \frac{x^2-5}{2x+3}$ b) $\lim_{x\to -1} \frac{3x^2+4x+1}{x+1}$ c) $\lim_{x\to 9} \frac{x-8}{\sqrt{x}-3}$
d) $\lim_{x\to 0} \frac{\tan(5x)}{\tan(6x)}$ e) $\lim_{x\to 1} \frac{|x^2-1|}{x-1}$

(4) Prove rigorously that $\lim_{x\to -1}(4+8x) = -4$.

(5) Find a, b so that $f(x)$ is continuous.

$$f(x) = \begin{cases} x^2 + a & x < 2 \\ b - x & -2 \leq x < 2 \\ 5 & x = 2 \\ cx - x^2 & 2 < x \end{cases}$$

(6) Complete the following sentences: The limit of $f(x)$ at a exists if ______. $f(x)$ is continuous at a if ______. $f(x)$ is differentiable at a if ______.

(7) Compute the derivative of the following functions from the definition.
a) $f(x) = 4 - x^2$ b) $g(x) = \frac{1}{2-x}$ c) $h(x) = x^3$

(8) Find the derivative with respect to x.
a) $y = \sin(2x)\cos^2 x$ b) $y = \frac{e^x}{x^2+1}$ c) $y = \arctan(x^3+1)$
d) $y = x^{\sqrt{x}} x^{\ln x}$ e) $\int_5^{\ln x} \sqrt{1-t^2}\, dt$

(9) Find the equation of the tangent line of $y/x = x + y$ at $x = 3$.

(10) You are traveling in a rocket which is traveling vertically at a speed of 800mph. The rocket is tracked through a telescope by your professor which is located 10 miles from the launching pad. Find the rate at which the angle between the telescope and the ground is increasing 3 minutes after lift-off.

(11) Estimate using linear approximation/ linearization $8.1^{1/3} - 2$.

(12) Find the linearization of $A(r) = 4/3\pi r^3$ at $a = 3$.

(13) Prove that $\sin x - \cos x = 3x$ has exactly one solution.

(14) Sketch the graph of the following functions (find min/max, inflection points, asymptotes, etc): a) $y = \frac{x}{x^3+1}$ b) $y = (x^2 - x)e^{-x}$

(15) Evaluate the limits.
a) $\lim_{x\to\infty} \frac{x^3+2x}{4x^3-9}$ b) $\lim_{x\to-\infty} \frac{12x+1}{\sqrt{4x^2+4x}}$ c) $\lim_{x\to 0} \frac{e^x-1}{\sin x}$
d) $\lim_{x\to 1}(1 + \ln x)^{1/(x-1)}$

(16) A box is constructed out of two different types of wood. The wood for the square top and bottom cost \$1 per square foot and the rectangular sides cost \$ 2 per square foot. Find the dimensions that minimize the cost if the box has volume V cubic feet.

(17) Use Newton's Method to find a root of $f(x) = x^2 - x - 1$ to two decimal places.

(18) Find the absolute maximum and absolute minimum of $f(x) = 3x^4 - 4x^3 - 12x^2 + 7$ on $[-2, 3]$.

(19) Find the indefinite integrals.
a) $\int (6x^7 + 4x^6 + 3x^2)\, dx$ b) $\int (y + 2)^4\, dy$ c) $\int x(x + 1)^{1/4}\, dx$
d) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

(20) Evaluate the definite integrals.
a) $\int_1^4 r^{-2}\,dr$ b) $\int_0^{\pi/4} \sec t \tan t\,dt$ c) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^4}}$

(21) a) Evaluate the Riemann sum for the function $f(x) = x^2$ on the interval $[0, 6]$ using six rectangles and right endpoints (i.e. R_6).
b) Find R_N.
c) Find $\int_0^6 x^2\,dx$ by finding $\lim_{N\to\infty} R_N$. Is it what you expected?

(22) Find the vertical displacement over the time interval $[1, 6]$ of a helicopter whose vertical acceleration at time t is $a(t) = 2t + 1$ and initial velocity is 0.

(23) Note: $P(t) = P_0 e^{kt}$. Find the decay constant of Radium-226 given that its half-life is 1622 years.

(24) Find the area bounded by $y = 4 - x^2$ and $y = x^2 - 4$.

Good luck on the final!