Practice Examination for Second Hour Examination

Mathematics 151, Fall 2007

1. Let $f(x) = x^{1/3}$.

(a) Find the linearization, $L(x)$, of $f(x)$ at $a = 8$.

(b) Use the linearization to approximate $7^{1/3}$ and $9^{1/3}$.

2. Let $f(x) = 2\sin(x) + \cos(2x)$.

(a) Find the critical points of f in the interval $[0, 2\pi]$.

(b) Find the maximum and minimum values of f on the interval $[0, 2\pi]$.

3. Calculate the following limits. In these k is a positive numerical constant.

(a) $\displaystyle\lim_{x\to 1} \frac{x^k - kx + k - 1}{(x-1)^2}$ (b) $\displaystyle\lim_{x\to 0^+} (1 + kx)^{1/x}$ (c) $\displaystyle\lim_{x\to\infty} (\ln(2e^x + x^k) - x)$

4. Determine the derivative of each of the following functions.

(a) $f(x) = x^3 \sin^2(2^x)$ (b) $f(x) = \tan^{-1}(e^x)$ (c) $f(x) = (1 + x^2)^{\sin(x)}$

5. Sketch the graph of each of the functions, determining exact algebraic answers for the critical points and inflection points and any horizontal or vertical asymptotes. In addition, determine which of the critical points are local maxima, local minima, or neither.

(a) $f(x) = x^2 e^{-x}$ (b) $f(x) = (x^2 - 1)^3$

6. Suppose that f is a differentiable function with $f(0) = 10$ and that for all $x \geq 0$, $f'(x) \leq -2$.

(a) Show that f must have a positive root, i.e., there must be some $x > 0$, so that $f(x) = 0$.

(b) What is the smallest $a > 0$ so that a root can be guaranteed to occur in the interval $[0, a]$?

(c) Is it possible for the function f to have two positive roots?

Explain your answers.

7. Let $f(x) = x^3 + x - 1$.

(a) Suppose that Newton's method is used to approximate a root of f. Give the recursive formula that expresses x_{n+1} in terms of x_n.

(b) If the initial approximation is $x_0 = 1$, find x_1 and x_2.

8. A function $y = f(x)$ is defined implicitly by the equation $2y + \sin(x + y) = 2x$.

(a) Find dy/dx in terms of x and y.

(b) Find the equation of the tangent line to the curve at the point $x = \pi/2$, $y = \pi/2$.

(c) Verify that f is increasing for all x.

9. A man 6 ft. tall walks away from a lamp that is 15 ft. above the ground with a speed of 3 ft./sec. When the man is 12 ft. from the lamppost,

(a) How long is his shadow?

(b) How fast is the length of his shadow increasing?

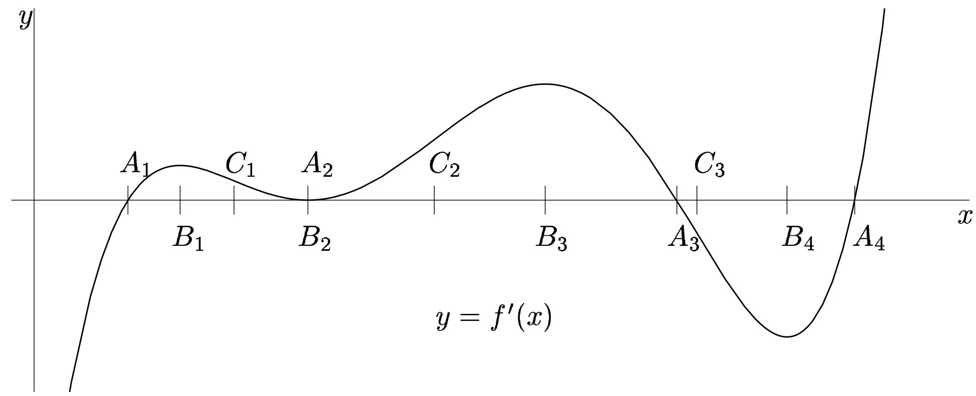
10. A rectangle with sides parallel to the x-axis and the y-axis is inscribed in the ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Of all such rectangles, find the dimensions of the one with maximum area.

11. In the following f is a differentiable function. The graph of the derivative of $f, y = f'(x)$, appears below. In the picture, the x-intercepts of the graph $y = f'(x)$ occur at $x = A_1, A_2, A_3, A_4$. The critical points of f' occur at $x = B_1, B_2, B_3, B_4$. The inflection points of f' occur at $x = C_1, C_2, C_3$.

Assuming that $f(A_1) = 0$, sketch the graph of $y = f(x)$, including those of the points A_i, B_j, C_k that are relevant to the graph of $y = f(x)$.



12. Assume that s is a function of t such that:

$$\frac{d^2s}{dt^2} = 60t^3 - 12t \quad \text{and at } t = 0, \ s = 4, \ \frac{ds}{dt} = -3.$$

Find s as a function of t.