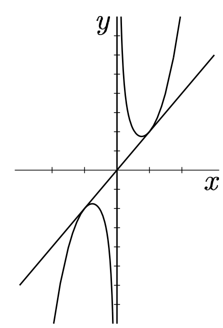
1(a) Domain: $[-4,4]$. Here $f(x)=\sqrt{4+x}+\sqrt{4-x}$ is defined if and only if $4+x\geq 0$ and $4-x\geq 0$. The first is equivalent to $x\geq -4$ and the second to $4\geq x$. Thus the domain is $[-4,4]$.
1(b) Domain: $[-2,2]$; Range: $[0,2]$. If $y=\sqrt{4-x^2}$, then $y\geq 0$, and squaring, $y^2=4-x^2$, or $x^2+y^2=4$. Thus, all points on the graph of $y=f(x)$ lie on the circle with center (0,0) and radius 2. Since also $y\geq 0$, the graph is the top half of this circle. Thus, the domain and range are as indicated.

2(a) 5 Since the limit is of type $0/0$, we factor $x-3$ from the numerator and denominator:
$$\lim_{x\to 3}\frac{3x^2-8x-3}{x^2-4x+3}=\lim_{x\to 3}\frac{(x-3)(3x+1)}{(x-3)(x-1)}=\lim_{x\to 3}\frac{3x+1}{x-1}=\frac{10}{2}=5.$$
2(b) 1/2 As the limit is of type $0/0$ and involves a square root, we multiply the numerator and denominator by the algebraic conjugate of the numerator:
$$\lim_{x\to 0}\frac{\sqrt{4+x}-\sqrt{4-x}}{x}=\lim_{x\to 0}\frac{(\sqrt{4+x}-\sqrt{4-x})(\sqrt{4+x}+\sqrt{4-x})}{x(\sqrt{4+x}+\sqrt{4-x})}=\lim_{x\to 0}\frac{(4+x)-(4-x)}{x(\sqrt{4+x}+\sqrt{4-x})}=$$
$$\lim_{x\to 0}\frac{2x}{x(\sqrt{4+x}+\sqrt{4-x})}=\lim_{x\to 0}\frac{2}{(\sqrt{4+x}+\sqrt{4-x})}=\frac{2}{2+2}=\frac{1}{2}.$$
2(c) 0 Here, $\lim_{x\to 0}\frac{1-\cos(2x)}{\sin(x)}=\lim_{x\to 0}\frac{(1-\cos(2x))/(2x)}{\sin(x)/(2x)}$. The limit of the numerator is: $\lim_{x\to 0}\frac{1-\cos(2x)}{2x}=$ $\lim_{u\to 0}\frac{1-\cos(u)}{u}=0$, found by replacing $2x$ by u, and using the fact that as x goes to 0, also u goes to 0. Since $\lim_{x\to 0}\frac{\sin(x)}{2x}=\frac{1}{2}$, the original limit is $\frac{0}{1/2}=0$.

3. In this problem, $f(x)=1/\sqrt{x}$. Then, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\left(\frac{1}{h}\right)\left(\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}\right)=$
$$\lim_{h\to 0}\left(\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}\right)=\lim_{h\to 0}\left(\frac{(\sqrt{x}-\sqrt{x+h})}{h\sqrt{x}\sqrt{x+h}}\cdot\frac{(\sqrt{x}+\sqrt{x+h})}{(\sqrt{x}+\sqrt{x+h})}\right)=\lim_{h\to 0}\left(\frac{x-(x+h)}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right)=$$
$$\lim_{h\to 0}\left(\frac{-h}{h\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right)=\lim_{h\to 0}\left(\frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}\right)=\frac{-1}{\sqrt{x}\cdot\sqrt{x}\cdot 2(\sqrt{x})}=-\frac{1}{2x^{3/2}}.$$

4. The graph of $y=f(x)=x^3+\frac{1}{x}$ together with the tangent line at $x=1$, whose equation is calculated in part (b) below appears at the right.



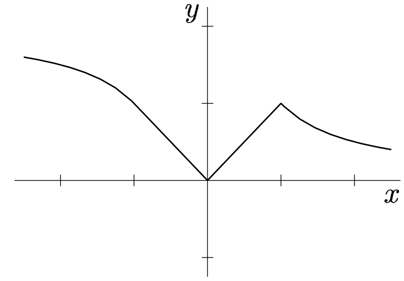
4(a) Since $y=f(x)=x^3+\frac{1}{x}$, $\frac{dy}{dx}=f'(x)=3x^2-\frac{1}{x^2}$ If the tangent line is horizontal for a given value of x, then $\frac{dy}{dx}=0$, or $3x^2-\frac{1}{x^2}=0$. Thus, $3x^4-1=0$, or $3x^4=1$. Thus, $x=\pm 1/3^{1/4}$, or equivalently $x=\pm 3^{-1/4}$.
4(b) At $x=1$, $y=2$, and $dy/dx=3-1=2$. Thus, the equation of the tangent line is $y-2=2(x-1)$, or $y=2x$.

5(a) $\frac{d}{dx}(\sqrt{x^4+4x+4})=\frac{d}{dx}\left((x^4+4x+4)^{1/2}\right)=(1/2)(x^4+4x+4)^{-1/2}(4x^3+4)=\frac{2(x^3+1)}{\sqrt{x^4+4x+4}}.$
5(b) $\frac{d}{dx}(x^2\sin^3(x^4))=\frac{d}{dx}(x^2)\cdot\sin^3(x^4)+x^2\frac{d}{dx}(\sin^3(x^4))=2x\sin^3(x^4)+x^2(3\sin^2(x^4))\frac{d}{dx}(\sin(x^4))=$ $2x\sin^3(x^4)+x^2(3\sin^2(x^4))(\cos(x^4)\cdot 4x^3)=2x\sin^3(x^4)+12x^5\sin^2(x^4)\cos(x^4).$
5(c) $\frac{d}{dx}\left(x^2e^{-x^3}\right)=\frac{d}{dx}\left(x^2\right)e^{-x^3}+x^2\frac{d}{dx}\left(e^{-x^3}\right)=2x\,e^{-x^3}+x^2(e^{-x^3}(-3x^2))=(2x-3x^4)e^{-x^3}.$

6. Let $y=f(x)=e^x/(e^x+1)$. Then, $(e^x+1)y=e^x$, or $e^xy+y=e^x$. Thus, $e^x-e^xy=e^x(1-y)=y$, or $e^x=y/(1-y)$. Taking ln( ) of both sides, $x=\ln(y/(y-1))$. Thus, $g(y)=\ln(y/(y-1))$, or $g(x)=\ln(x/(x-1))$, or $g(x)=\ln(x)-\ln(x-1)$.

7. Since $\log_u(5)=a$, $\log_u(27)=b$, $\log_u(32)=c$, $u^a=5$, $u^b=27$, and $u^c=32$. Thus, it follow that: $u^{2a+(1/3)b-(2/5)c}=u^{2a}u^{(1/3)b}u^{(-2/5)c}=(u^a)^2(u^b)^{1/3}/(u^c)^{2/5}=5^2(27)^{1/3}/(32)^{2/5}=5^2 3/4=75/4.$

8. Here, $f(1) = 2, f'(1) = 4$ and, $g(x) = x^4 - x + 1, g'(x) = 4x^3 - 1$. Thus, $g(0) = g(1) = 1$, $g'(0) = -1, g'(1) = 3$. (a) $(fg)'(1) = f(1)g'(1) + g(1)f'(1) = 2 \cdot 3 + 1 \cdot 4 = 10$ (b) $(f/g)'(1) = (g(1)f'(1) - f(1)g'(1))/g(1)^2 = (1 \cdot 4 - 2 \cdot 3)/1^2 = -2$. (c)$(f \circ g)(0) = f(g(0)) = f(1) = 2$ (d) $(f \circ g)'(0) = f'(g(0))g'(0) = f'(1)g'(0) = 4 \cdot -1 = -4$ (e)$(f \circ g)(1) = f(g(1)) = f(1) = 2$ (f) $(f \circ g)'(1) = f'(g(1))g'(1) = f'(1)g'(1) = 4 \cdot 3 = 12$



9(a) Since f is to be continuous, the left and right hand limits at each point must be equal. At $x = -1$, the left hand limit is $-1 + A$ and the right hand limit is 1, and so $-1 + A = 1$, or $A = 2$. Evaluating the left and right hand limits at $x = 1$ gives $1 = 1 + B$, or $B = 0$. Thus, $A = 2, B = 0$. 9(b) The graph appears to the right.
9(c) The function f fails to be differentiable at $x = 1$ and $x = 0$, since the graph has cusps at these points. More precisely, if $f'(1)$ exists, then $f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$. Now to the left of $x = 1$, and near $x = 1$, $f(x) = |x| = x$, since $x > 0$. Thus, the left hand limit, $\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$, is the derivative of x at $x = 1$, and so is 1. Similarly, the right hand limit, $\lim_{h \to 0^+} \frac{f(1+h) - f(1)}{h}$, is the derivative at $x = 1$ of $1/x + B = 1/x$. Thus, it is $-1/x^2$, evaluated at $x = 1$, i.e., is -1. Since the left and right hand limits differ, the derivative $f'(1)$ does not exist. Similarly, at $x = 0$ the derivative $f'(0)$ does not exist.

10(a) Let $h(x) = 3^x - x^3$. Then, $h(2.4) = 13.97 - 13.82 = .15$ and $h(2.5) = 15.59 - 15.62 = -.03$. Since h changes sign at the endpoints of the interval $[2.4, 2.5]$, $h(r) = 0$ for some r in the interval $[2.4, 2.5]$. Thus, $3^r = r^3$.
10(b) Let $h(x) = 2^x - x^3$. Then, $h(1) = 2 - 1 = 1 > 0$ and $h(2) = 2^2 - 2^3 = -4 < 0$. Thus, h has a root and $2^x = x^3$ a solution in in the interval $[1, 2]$. Likewise, $h(9) = 2^9 - 9^3 = 512 - 729 < 0$, $h(10) = 2^{10} - 10^3 = 1024 - 100 > 0$, and $2^x = x^3$ has a solution in in the interval $[9, 10]$.
11(a) Let the distance of the arrow above the ground be $s(t)$, measured in feet. Then, $s(t) = s_0 + v_0 t - 16t^2$, where s_0 is the initial position and v_0 is the initial velocity. Since the arrow is initially at ground level, $s_0 = 0$. Thus, $s(t) = v_0 t - 16t^2$. Also, the arrow returns to the ground when $s(t) = v_0 t - 16t^2 = 0$, or $t = v_0/16$. Since this 6 seconds after the arrow is shot, $v_0/16 = 6$, or $v_0 = 96$ ft/sec.
11(b) Since $s(t) = 96t - 16t^2$, and the arrow is highest at the midpoint of its flight at $t = 3$, the arrow goes up to $s(3) = 144$ ft.
12(a) $f'(x) = 2\cos x + 2\sin x \cos x = 2\cos x(1 + \sin x)$.
12(b) Since $f(\pi) = 0$ and $f'(\pi) = 2\cos \pi = -2$, the equation of the tangent line is $y - 0 = -2(x - \pi)$, or $y = -2x + 2\pi$.
12(c) The tangent line to the graph of $y = f(x)$ will be horizontal if $f'(x) = 0$. Thus, $2\cos x(1 + \sin x) = 0$, and so either $\cos x = 0$ or $1 + \sin x = 0$. In the first case $x = (2n+1)\pi/2$, with n any integer. In the second case, $\sin x = -1$, and $x = 3\pi/2 + 2k\pi$, with k an integer. Thus, the second case answer is also an odd integer times $\pi/2$, and is of the first type. Thus, $x = (2n+1)\pi/2$, with n any integer.

13. In this problem, $\frac{dy}{dx} = 2x - 1$. Since (a, b) is on the parabola $y = x^2 - x$, $(a, b) = (a, a^2 - a)$. The slope of the tangent line at point (a, b) is $2a - 1$. Thus, the equation of the tangent line at (a, b) is $y - (a^2 - a) = (2a - 1)(x - a)$. Since (2,1) lies on this line, $1 - (a^2 - a) = (2a - 1)(2 - a)$, or $-a^2 + a + 1 = -2a^2 + a + 4a - 2$, or $a^2 - 4a + 3 = (a - 1)(a - 3) = 0$. Thus, $a = 1$ or $a = 3$. Thus, $(a, b) = (1, 0)$ or (3,6).

14. In the first graph, $y = f(x)$, the graph crosses the x-axis in 4 places and has a horizontal tangent line in 5 places. It follows that f has 4 roots and f' has 5 roots.

In the second graph, $y = g(x)$, the graph crosses the x-axis in 3 places and has a horizontal tangent line in 4 places. It follows that g has 3 roots and g' has 4 roots.

Then $f' \neq g$, since f' and g have a different number of roots. Thus, $g' = f$, i.e., the first graph is the graph of the derivative of the function in the second graph.
15. Since $f(x) = e^{2x^2}$, $f'(x) = e^{2x^2}(4x)$, and $f''(x) = e^{2x^2}(4x)^2 + e^{2x^2}(4) = e^{2x^2}(16x^2 + 4)$.