Practice Examination for First Hour Examination

Mathematics 151, Fall 2007

1. Find the domain of both functions and the range of the second function:

(a) $f(x) = \sqrt{4+x} + \sqrt{4-x}$ (b) $f(x) = \sqrt{4-x^2}$

2. Determine each of the limits:

(a) $\lim_{x\to 3} \dfrac{3x^2-8x-3}{x^2-4x+3}$ (b) $\lim_{x\to 0} \dfrac{\sqrt{4+x}-\sqrt{4-x}}{x}$ (c) $\lim_{x\to 0} \dfrac{1-\cos(2x)}{\sin(x)}$

3. Use the definition of the derivative directly to calculate $f'(x)$, where $f(x) = 1/\sqrt{x}$.

4. Sketch the graph of $y = f(x) = x^3 + \dfrac{1}{x}$.

(a) Determine those points x so that the tangent line to the curve at x is horizontal. Give exact values, not numerical approximations.

(b) Find the equation of the tangent line to the curve at the point $x = 1$ and draw the tangent line on the earlier graph.

5. Calculate the derivative of each fuction f:

(a) $f(x) = \sqrt{x^4+4x+4}$ (b) $f(x) = x^2\sin^3(x^4)$ (c) $f(x) = x^2e^{-x^3}$

6. Find the inverse function g of the function $f(x) = e^x/(e^x+1)$.

7. Suppose that u is a positive real number and a, b, c are real numbers so that:

$\log_u(5) = a$, $\log_u(27) = b$, $\log_u(32) = c$. What is the numerical value of $u^{2a+(1/3)b-(2/5)c}$?

8. Suppose that f and g are functions. Asume that f is differentiable and $f(1) = 2, f'(1) = 4$ and that $g(x) = x^4 - x + 1$. Use this information to calculate:

(a) $(fg)'(1)$ (b) $(f/g)'(1)$ (c) $(f\circ g)(0)$ (d) $(f\circ g)'(0)$ (e) $(f\circ g)(1)$ (f) $(f\circ g)'(1)$

9. With A and B constants, a function f is defined by: $f(x) = \begin{cases} 1/x + A, & \text{if } x < -1; \\ |x|, & \text{if } -1 \le x \le 1; \\ 1/x + B, & \text{if } 1 < x. \end{cases}$

(a) Find A and B so that f is continuous everywhere.

(b) Sketch the graph of $y = f(x)$.

(c) At which points does f fail to be differentiable? Explain the answer.

10. Let $f(x) = 3^x$ and $g(x) = x^3$. Then, $f(3) = 3^3 = g(3)$.

(a) Use the following table to show that $3^x = x^3$ has another solution and find an interval of length 0.1 in which the solution lies.

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
3^x	9	10.05	11.21	12.51	13.97	15.59	17.4	19.42	21.67	24.19	27
x^3	8	9.26	10.65	12.17	13.82	15.63	17.08	19.68	21.95	24.39	27

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(b) If $f(x) = 2^x$ and $g(x) = x^3$, show that the equation $f(x) = 2^x = x^3 = g(x)$ has at least two solutions.

11. An arrow is shot straight up from ground level and stays in the air for 6 seconds.

(a) What is the initial velocity of the arrow?

(b) How high does the arrow go?

12. Let f be the function defined for all x by $f(x) = 2\sin x + \sin^2 x$.

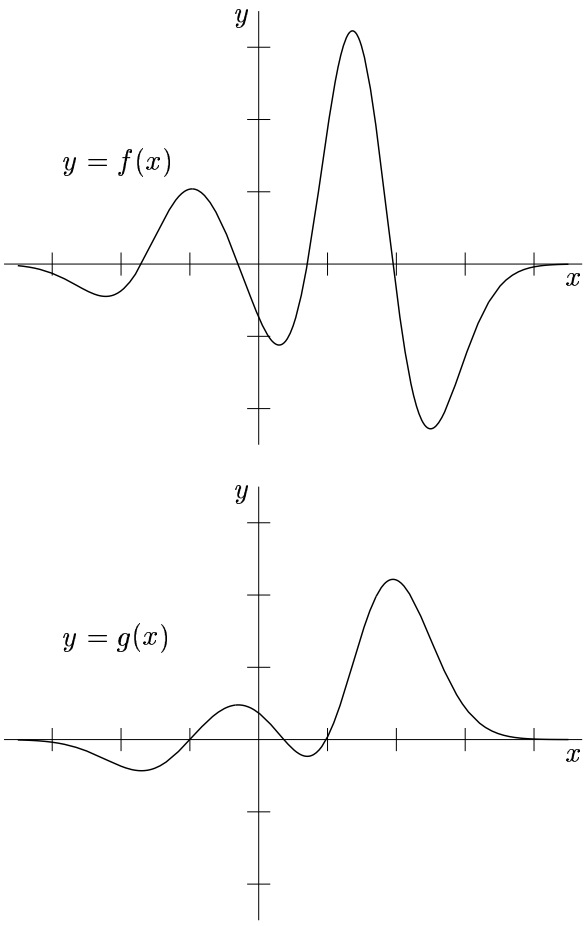
(a) Find $f'(x)$.

(b) Find the equation of the tangent line of $y = f(x)$ at $x = \pi$.

(c) Find all values of x for which the tangent line of $y = f(x)$ is horizontal.

13. Find all points (a, b) on the parabola $y = x^2 - x$ so that the tangent line to the parabola at the point (a, b) contains the point $(2, 1)$.

14. Below are the graphs of two functions f and g. One of the functions is the derivative of the other. Determine which is the original function and which is its derivative, explaining your reasons.



15. If $f(x) = e^{2x^2}$, find the second derivative , f'', of f.