Extra Problem. As at the problem about sages each of alchemist tries to imagine what think other ones; after the order some of them know 7 on dishonest servants, some 6.

What do think the last ones?

What is clear?

If somebody didn't know any traitor then his servant is traitor and he would be kill him; the if somebody was killed at 1st night or 2nd night other alchemists who know only 1 traitor will be kill his one etc.

All 7 traitor will be killed at 7th night by 7 alchemists who knows only 6 traitors. The number of alchemists isn't essential.
3) \[ A = (\forall x) (P(x) \lor Q(x)) \]
\[ B = (\forall y) (P(y) \lor \forall z Q(z)) \]

\[ B \Rightarrow A - \text{Tautology} \]
\[ (\forall y) P(y) \quad \text{thrift means} \quad P(y) \equiv T \]

\[ B \text{ means at least one of them is tautology but for their disjuncts is True} \]

But if \( A \) is True, it means that for each \( x \), \( P(x) \) or \( Q(x) \) is True but it doesn't mean that one of the at least of them is tautology.

Typical example:

\[ \Sigma \text{INJ} P(x) : x \text{ is odd}; \quad Q(x) : x \text{ is odd-even} \]

Then \( (\forall x) P(x) \lor Q(x) \) is True but \( (\forall y) P(y) \) and \( (\forall z) Q(z) \) are False.
Axiomatic theories
1. Fix subject fields
2. Denote and give names basic predicates
3. New predicates defined through basic ones
4. Some closed formulas are taken as true one (axioms or postulates)
5. Other theorems are the consequences of axioms

First example: Euclidean geometry on the plane, (IV cent. BC)

Basic subject fields:
- Points \( P \)
- Lines \( L \)

Basic predicates:
1. \( x \in l \) \( (x \text{-point, } l \text{-line}) \) (point on the line \( l \))
2. \( x < y < z \) \( \) (point \( y \) lies between \( x \) and \( z \))

More exact:
\( (\exists l) \) \( x \in l \land y \in l \land z \in l \) (only if)
New predicates

3. Interval \( \exists (x, y) \in \mathbb{R} \times x < z < y \).

4. \( x \) is the intersection point of \( l, m \)
   \( x \in l \wedge x \in m \)

5. \( l \perp m \)

6. \( l \parallel m \)

7. Triangle \( \triangle x y z \)
   \( (x, y) \equiv (u, v) \) - congruency

Axioms

1. \( (\forall l) \ (\exists x) (\exists y) \ x \in l \wedge y \in l \wedge x \neq y \)
   (on each line there are at least 2 different points)

2. \( (\forall l) \ (\exists x) \forall x \in l \)
   (for each line there is a point outside the line)

3. \( (\forall x) (\forall y) (\forall u) (\forall v) \ (x, y) \equiv (u, v) \wedge (u, v) \equiv (s, t) \)
   (transitivity of congruency)

4. \( (\forall x) (\forall y) (\forall z) (\forall l) (x \in l \wedge y \in l \wedge z \in l) \Rightarrow \exists l 

(\forall x) (\forall y) (\forall l) x \in l \wedge y \in l \wedge z \in l \Rightarrow \exists l

(\exists z) z \in l \wedge x < z < y \).
\[ (\forall x)(\forall y) \ x \not= y \Rightarrow (\exists ! \ c) \ x \in c \land y \in c \]

There is a line

For through 2 different \( P \) points \( \Rightarrow \) there is a line passing through these points and this line is unique.

Theorem. Two different lines can intersect not more than 1 point.

\[ (\forall c)(\forall m) \ c \not= m \Rightarrow ((\forall x)(\forall y) \ (x \in c \land y \in c \land x \not= y) \land (x \in m \Rightarrow x = y)) \]

Proof. By contradiction. \(~Q \Rightarrow (\exists x)(\exists y) \ x \in c \land y \in c \land x \not= y\)

Definition. \( (\text{Intersection point}) \) \( \Rightarrow \) Contradiction

\[ x = c \cap m = x \in c \land x \in m \]

Proof, contradiction.

\[ \sim Q : (\exists x)(\exists y) \ x \in c \land y \in c \land x \not= y \]

Axiom: \( (\exists ! \ c) \ x \in c \land y \in c \)

Contradiction: \( c \not= m \)

5th postulate. \( (\forall c)(\forall x) \ x \in c \Rightarrow (\exists m) \ x \in m \) with \( \text{axiom} \)

\[ (\forall x)(\forall c) \ x \in c \Rightarrow (\exists m) \ x \in m \]
Let \( l(x, y), x \neq y \) be the unique line passing through \( x, y \).

There are several axioms for each basic predicate:

"lie between":

\[
(\forall x)(\forall y)(\forall z)(x < y < z \Rightarrow \neg (\exists z \in l(x, y) : y < z < x)).
\]

**Corollary:**

\[
(\forall x)(\forall y)(\forall z) (x < y, z \in l \Rightarrow (\exists ! w)(x, y, z, w) \text{ which lies between } x \text{ and } y).
\]

It's possible to write by a formula

\[
(\forall x)(\forall y)(\forall l) x \in l \land y \in l \Rightarrow (\exists z)(\exists u) \land x < z < y \land x < y < u.
\]

**Corollary:** There is \( \infty \) set of points on any line.
\[ (\forall x)(\forall y) \ x \neq y \Rightarrow (\exists ! l) \ x \in l \land y \in l \]

There is a line

For

Through 2 different \( P \) points there is a line passing through these points and this line is unique.

Theorem. Two different lines can intersect not more than 1 point.

\[ (\forall l)(\forall m) \ l \neq m \Rightarrow ((\forall x)(\forall y) (x \in l \land y \in m \land x \neq y)) \]

Definition. \( x \) (Intersection point).

\[ x = l \cap m = x \in l \land x \in m \]

Proof. By contradiction. \( \neg Q \Rightarrow (\exists x)(\exists y) x \in l \land y \in m \land x \neq y \]

Definition. \( x \) (Intersection point).

Contradiction: \( l \neq m \)

5th postulate. \( (\forall l)(\forall x) (x \in l \Rightarrow (\exists m) x \in m \land l \neq m) \) (axiom)