Lecture 10.

1. Quiz 4.
2. HA 5.
3. Set 1.7.

2. a) 2 cases

- $n$ even $\Rightarrow n^2, n, 5n^2 + 3n, 5n^2 + 3n + 4$ are even
- $n$ odd $\Rightarrow 5n^2, 3n$ odd $\Rightarrow 5n^2 + 3n$ even

b) Possible remainders for 5 are 0, 1, 2, 3, 4. Their sum is multiple 5.
Remainders of 5 consecutive number 0, 1, 2, 3, 4, starting from one of them.
So their sum is multiple 5.

(c) $n^3 - 4n = n(n-1)(n+1)$ - product of 3 consecutive number.
Between them one multiple 3 and at least one is even $\Rightarrow 6|n^3 - 4n$

(d) (not in HA)

$12| (n^3 - n)(n+2)$. Between $n, n+1, n+2$, will there be even and one of them multiple 4.
4. (a) Contradiction

\[(p \land \lnot q) \Rightarrow \exists \ l \ q \land x+y - \text{rational} \]

Contradiction.

(b) Take any rational \( z \) and irrational \( x \), and \( z \approx x \), \( y = z - x \). Then \( y \) is irrational, \( z = x + y \). (More details!)

\[7a, \ x > 0 \Rightarrow \frac{12x-1}{x+1} \leq 2\]

\[x > 0 \Rightarrow \text{equiv.} \quad \begin{cases} 2x \leq 12x - 1 \leq 2(12x+1) \end{cases} (x)\]

2 cases: 1) \( x \geq \frac{1}{2} \Rightarrow 12x - 1 = 2x - 1 \)

\[\Rightarrow \quad 2x - 1 \leq 2x + 2 \quad \Rightarrow -1 \leq 2\]

2) \( x < \frac{1}{2} \Rightarrow 12x - 1 = -2x - 2 + 1\)

\[\Rightarrow \quad -2x - 1 \leq 2x + 2\]

For \(-x > 0\)
9b) \( b = aq + r, 0 \leq r < a \)

\[ a = 5, \quad b = 36 \]

\[ 36 = 5q + r \implies q = 7, \quad r = 1 \]

Sect 1.3.

1. (c) \( P(x) \) - \( x \) is isosceles
   \( Q(x) \) - \( x \) is \( \bigtriangleup \) right

\[ \exists x \left( P(x) \land Q(x) \right) \]

(a) \( \forall x \left( Q(x) \implies P(x) \right) \).

(f) \( P(x) \) - \( x \) is honest

\[ (\exists x \ P(x)) \land (\forall y \ P(y)) \]

(g) \( \forall x \ x + 0 \equiv (x > 0 \lor x < 0) \)

(w) \( R(x, y) \) - \( x \) loves \( y \)

\[ \forall x \ \exists y \ R(x, y) \]
Extra problems.

1. a) \( p = 3 , b = 1 : 3 \mid 9 \)
   \( p = 11 , b = 2 : 11 \mid 99 \)
   \( p = 37 , b = 3 : 37 \mid 999 \)

b) \( \frac{a_k}{a_o} \frac{a}{a_k} \)

   \( = a_0 + a_1 10 + \cdots + a_k 10^k \)

\[ 10^m = 3q_3 + 1 \]
\[ 10^m = 9q_9 + 1 \]

So for 3, 9 remainder of equal
remainder of \( a_0 + a_1 + \cdots + a_n \).

C) Let \( m = \overline{a_2 a_o} + \overline{a_3 a_2} + \cdots + \overline{a_k a_{k-1}} \)

   \( h = \overline{a_2 a_o} + \overline{a_3 q_2 \cdot 100 + \overline{a_5 q_4 \cdot 100^2} + \cdots} \)

Remainder \( m, h \) in the division on 11

Coincide.

Proof: \( 11|10^m \) \( \iff 11|100^s - 1 \)

2) Last digit must be even so it is 8.
First digit can't be 1 since \( 183 > 138 \).

So \# of last page 86 is 318.

\[ 318 - 183 - 50 + \text{p} = 318 - 132 = 186 \]