MATH 300. INTRODUCTION TO MATHEMATICAL REASONING.
FALL 2015.
WEEK 9 (LECTURE 16-17).
PREDICATS AND QUANTIFIERS. REVIEW FOR MT2.

1. Reading: Sections 1.3, 1.6 and Lecture Notes.
2. Home assignment (Due Monday, November 2) (to submit). 1. Prove Tautologies

\[(\forall x)(P(x) \land Q(x)) \iff (\forall y)P(y) \land (\forall z)Q(z);\]

\[(\exists x)(P(x) \lor Q(x)) \iff (\exists y)P(y) \lor (\exists z)Q(z);\]

\[(\exists x)(P(x) \land Q(x)) \Rightarrow (\exists y)P(y) \land (\exists z)Q(z);\]

\[(\forall x)(P(x) \Rightarrow Q(x)) \Rightarrow ((\forall y)P(y) \Rightarrow (\forall z)Q(z));\]

but the following formulas are not tautologies (construct counterexamples):

\[(\exists x)(P(x) \land Q(x)) \iff (\exists y)P(y) \land (\exists z)Q(z);\]

\[(\forall x)(P(x) \Rightarrow Q(x)) \iff ((\forall y)P(y) \Rightarrow (\forall z)Q(z)).\]

2. Let us \(N\) is the product of 10 consequent natural number. Find maximal \(a, b, c\) such that all such \(N\) we have

\[2^a3^b5^c|N.\]

3. Using axioms prove that the set of points on any line is infinite; that there is infinite number points outside of any line.

4. Give a predicative definition of a parallelogram \(xyuv\). Prove that if the points \(x, y, u\) do not lie on one line then there is a parallelogram with these vertexes and such parallelogram is unique.

5. Prove that the product of 2 real numbers is negative iff exactly one of them negative.

6. Prove that \(-x^2 - 2x + 15 > 0\) iff \(-5 < x < 3.\)