Homework grading policy:

1. 5 problems will be chosen for grading each time.
   2 of them will be chosen for presentation.
   The list will be announced at the beginning of class.

2. Every problem will be graded on a 2 pt scale.
   2pt means satisfactory, 1pt means unsatisfactory
   and 0pt means very unsatisfactory.

3. Late homework submission is allowed up to Friday 11:59 PM.
   Even later submissions would suffer penalty of 2pts

4. If you got a grade < 8pt, by submitting make-up work you can
   make it to 8pts, provided all your solns are satisfactory.

Some addition policies will be announced on sakai.
Review: What you should have learned in Chap 1.

1. To represent lengths of segments, \(\mathbb{R}\) is not enough.
2. Manipulation of sets and functions.
3. Prove by contradiction and prove by induction.
4. Least upper bound, Greatest lower bound, Axiom of Completeness.
5. Nested Interval Property.
6. Archimedeon Property: \(\forall x \in \mathbb{R}, \exists n \in \mathbb{N}, n > x\)
   \(\forall y > 0, \exists n \in \mathbb{N}, \frac{1}{n} < y\)

Density of \(\mathbb{Q} \subseteq \mathbb{R}\): \(\forall a, b \in \mathbb{R}, a < b \Rightarrow \exists r \in \mathbb{Q}, a < r < b\).

7. 1-1 correspondence between sets.
   Countable sets. Uncountable sets.
   Nested Interval Property or \(\mathbb{J} \Rightarrow \mathbb{R}\) is uncountable.
   Cantor's diagonalization method.

HW to be graded: 1.39, 1.4.2, 1.4.4*, 1.6.2*, 1.6.10b.
For Chap 2:

Sequence, Convergence of a sequence. Uniqueness of a limit.

(1) Every seq. is bounded.

(2) Algebraic Limit Thm. \((c_{an}, an \rightarrow b, \frac{a_n}{b_n} \rightarrow b, x \rightarrow o)\)

(3) If \(a_n \rightarrow a, \ b_n \rightarrow b\), then
   (i) \(\forall n, a_n \rightarrow o \Rightarrow a \rightarrow o\)
   (ii) \(\forall n, a_n \leq b_n \Rightarrow a \leq b\).
   (iii) If \(\exists c \in \mathbb{R}, c \leq b_n, \forall n \Rightarrow c \leq b\).
       \(\exists c \in \mathbb{R}, a_n \leq c, \forall n \Rightarrow a \leq c\).

My solution to 1.4.4.

Recall: \(b = \sup T\) if
   (i) \(\forall x \in T, x \leq b\)
   (ii) \(\forall b', b < b', \exists x \in T, x > b'\)

Since \(T = [a, b] \cap \mathbb{Q}\), (i) is obviously satisfied.

To see (ii), simply notice from density of \(\mathbb{Q}\) in \(\mathbb{R}\), that
   \(\forall b' < b, \exists x \in \mathbb{Q}, b' < x < b\).

Without loss of generality, \(x\) can be chosen in \([a, b]\).

[why? If \(b' < a\), simply pick \(x\) between \(a\) and \(b\).]

[If \(b' < a\), then \(x\) is automatically \([a, b]\).]

So \(x \in \mathbb{Q} \cap [a, b] = T\). Then (ii) is proved.
My solution to 1.6.2.

Recall: \( f : N \to (0, 1) \)

\[
\begin{align*}
f(1) &= 0, a_{11} a_{12} \ldots a_{1n} \ldots = a_{11} \cdot 10^{-1} + a_{12} \cdot 10^{-2} + \ldots + a_{1n} \cdot 10^{-n} + \ldots \\
f(2) &= 0, a_{21} a_{22} \ldots a_{2n} \ldots = a_{21} \cdot 10^{-1} + a_{22} \cdot 10^{-2} + \ldots + a_{2n} \cdot 10^{-n} + \ldots \\
&\vdots \quad \vdots \quad \vdots \quad \vdots \\
f(n) &= 0, a_{n1} a_{n2} \ldots a_{nn} \ldots = a_{n1} \cdot 10^{-1} + a_{n2} \cdot 10^{-2} + \ldots + a_{nn} \cdot 10^{-n} + \ldots \\
&\vdots \quad \vdots \quad \vdots \quad \vdots
\end{align*}
\]

with all \( a_{ij}'s \) in \( \{0, 1, 2, \ldots, 9\} \).

Set \( x = 0, b_1 b_2 \ldots b_n \ldots = b_1 \cdot 10^{-1} + b_2 \cdot 10^{-2} + \ldots + b_n \cdot 10^{-n} + \ldots \)

with \( b_i = \begin{cases} 2 & \text{if } a_{ii} \neq 2 \\ 3 & \text{if } a_{ii} = 2 \end{cases} \)

So \( b_i \neq a_{ii}, \forall i \).

(1) Since \( b_1 \neq a_{11}, \ x \neq f(1) \)

(2) Since \( b_2 \neq a_{22}, \ x \neq f(2) \)

In general, since \( b_n \neq a_{nn}, \forall n \in N, \ x \neq f(n) \)

(3) So no matter how \( f : N \to (0, 1) \) is given, there always exists a number in \( (0, 1) \), \( x \neq f(n), \forall n \in N \).

More formally, \( \forall f : N \to (0, 1), \exists x \in (0, 1), \ x \neq f(N) \)

This shows any \( f : N \to (0, 1) \) cannot be surjective.

Therefore \((0, 1)\) is not countable.
Remark: The same argument can be used to show
\[ \{0,1\} \times \{0,1\} \times \cdots \times \{0,1\} \times \cdots \]
(Cartesian product of countably infinite copies of \(\{0,1\}\))

is uncountable. Just modify \(\ast\)

Remark: One can also show that
\[ \{0,\overline{1}\} \times \{0,\overline{1}\} \times \cdots \times \{0,\overline{1}\} \times \cdots \leftrightarrow (0,1) \]

\[ \Rightarrow \{0,\overline{1}\} \times \cdots \times \{0,\overline{1}\} \times \cdots \leftrightarrow \{0,1\} \times \{0,1\} \times \cdots \times \{0,1\} \times \cdots \]