What you should learn from Recitation 9: Laplace Transforms

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There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.
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Laplace Transform

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And we have the following properties

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For any numbers $k_1, k_2$,
$$\mathcal{L}(k_1 f_1(t) + k_2 f_2(t)) = k_1 \mathcal{L}(f_1(t)) + k_2 \mathcal{L}(f_2(t))$$

**Injectivity:**
If $f_1(t); f_2(t)$ are continuous and $\mathcal{L}(f_1(t)) = \mathcal{L}(f_2(t))$,
then $f_1(t) = f_2(t)$.

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  \[ \mathcal{L}(u_c(t)f(t - c)) = F(s), \]

  as showed in MIT Lecture 22.

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Derivative formulas

In order to use Laplace transform to solve ODEs, you should also memorize these formulas:

\[ L(f'(t)) = sF(s) - f(0) \]

\[ L(f''(t)) = s^2F(s) - sf(0) - f'(0) \]

\[ L(f'''(t)) = s^3F(s) - s^2f(0) - sf'(0) - f''(0) \]

\[ L(f''''(t)) = s^4F(s) - s^3f(0) - s^2f'(0) - sf''(0) - f'''(0) \]

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\mathcal{L}(f^{(3)}(t)) &= s^3 F(s) - s^2 f(0) - sf'(0) - f''(0) \\
\mathcal{L}(f^{(4)}(t)) &= s^4 F(s) - s^3 f(0) - s^2 f'(0) - sf''(0) - f'''(0)
\end{align*}
\]

Attention: Don’t mess up with the signs! From the second term on, everything is negative.
In order to use Laplace transform to solve ODEs, you should also memorize these formulas: Assuming $\mathcal{L}(f(t)) = F(s)$:

\[
\begin{align*}
\mathcal{L}(f'(t)) &= sF(s) - f(0) \\
\mathcal{L}(f''(t)) &= s^2F(s) - sf(0) - f'(0) \\
\mathcal{L}(f^{(3)}(t)) &= s^3F(s) - s^2f(0) - sf'(0) - f''(0) \\
\mathcal{L}(f^{(4)}(t)) &= s^4F(s) - s^3f(0) - s^2f'(0) - sf''(0) - f^{(3)}(0)
\end{align*}
\]
Derivative formulas

In order to use Laplace transform to solve ODEs, you should also memorize these formulas: Assuming $L(f(t)) = F(s)$:

\begin{align*}
L(f'(t)) &= sF(s) - f(0) \\
L(f''(t)) &= s^2F(s) - sf(0) - f'(0) \\
L(f^{(3)}(t)) &= s^3F(s) - s^2f(0) - sf'(0) - f''(0) \\
L(f^{(4)}(t)) &= s^4F(s) - s^3f(0) - s^2f'(0) - sf''(0) - f^{(3)}(0)
\end{align*}

Attention: Don’t mess up with the signs! From the second term on, everything is negative.
Find the solution to the following IVP:

\[ y'' - 3y' + 2y = e^t, \quad y(0) = 0, \quad y'(0) = 1 \]
Quiz Problem 2

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- Perform the Laplace transform:
Quiz Problem 2

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Perform the Laplace transform: Let \( Y(s) = \mathcal{L}(y(t)) \),
Quiz Problem 2

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(s^2 Y(s) - sy(0) - y'(0))
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Quiz Problem 2

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- Perform the Laplace transform: Let \( Y(s) = \mathcal{L}(y(t)) \), then

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(s^2 Y(s) - sy(0) - y'(0)) - 3(sY(s) - y(0))
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Putting in \( y(0) \) and \( y'(0) \):
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Putting in \( y(0) \) and \( y'(0) \):

\[ (s^2 - 3s + 2)Y(s) - 1 = \frac{1}{s - 1} \]
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Solve for \( Y(s) \):
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\[
(s^2 - 3s + 2)Y(s) - 1 = \frac{1}{s - 1}
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Solve for \( Y(s) \):

\[
Y(s) = \frac{s}{(s^2 - 3s + 2)(s - 1)}.
\]
Quiz Problem 2

- Break $Y(s)$ into partial fractions:
Quiz Problem 2

- Break $Y(s)$ into partial fractions:

$$Y(s) = \frac{s}{(s - 2)(s - 1)^2}$$
Quiz Problem 2

Break $Y(s)$ into partial fractions:

$$Y(s) = \frac{s}{(s - 2)(s - 1)^2} = \frac{A}{(s - 1)^2} + \frac{B}{s - 1} + \frac{C}{s - 2}$$
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By the cover-up method, one quickly determines that
Quiz Problem 2

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$$A = \left. \frac{s}{s - 2} \right|_{s=1}$$
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Now compute to determine $B$. 

---

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Quiz Problem 2

Break \( Y(s) \) into partial fractions:

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Y(s) = \frac{s}{(s - 2)(s - 1)^2} = \frac{A}{(s - 1)^2} + \frac{B}{s - 1} + \frac{C}{s - 2}
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A = \left. \frac{s}{s - 2} \right|_{s=1} = -1, \quad C = \left. \frac{s}{(s - 1)^2} \right|_{s=2} = 2.
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$$B = \frac{s}{s-1} = \frac{s}{(s-2)(s-1)^2} - \frac{1}{(s-1)^2} - \frac{2}{s-2}$$
Quiz Problem 2

- Break $Y(s)$ into partial fractions:

$$Y(s) = \frac{s}{(s - 2)(s - 1)^2} = \frac{A}{(s - 1)^2} + \frac{B}{s - 1} + \frac{C}{s - 2}$$

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$$\frac{B}{s - 1} = \frac{s}{(s - 2)(s - 1)^2} - \frac{-1}{(s - 1)^2} - \frac{2}{s - 2}$$

$$= \frac{s + (s - 2) - 2(s - 1)^2}{(s - 2)(s - 1)^2}$$
Quiz Problem 2

• Break \( Y(s) \) into partial fractions:

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Y(s) = \frac{s}{(s - 2)(s - 1)^2} = \frac{A}{(s - 1)^2} + \frac{B}{s - 1} + \frac{C}{s - 2}
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\[
\frac{B}{s - 1} = \frac{s}{(s - 2)(s - 1)^2} - \frac{1}{(s - 1)^2} - \frac{2}{s - 2}
\]

\[
= \frac{s + (s - 2) - 2(s - 1)^2}{(s - 2)(s - 1)^2} = \frac{2(s - 1)^2 - 2(s - 1)^2}{(s - 2)(s - 1)^2}
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$$= \frac{s + (s - 2) - 2(s - 1)^2}{(s - 2)(s - 1)^2} = \frac{2(s - 1) - 2(s - 1)^2}{(s - 2)(s - 1)^2}$$

$$= \frac{2 - 2(s - 1)}{(s - 2)(s - 1)}$$
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$$= \frac{2 - 2(s - 1)}{(s - 2)(s - 1)} = \frac{-2s + 4}{(s - 2)(s - 1)}$$
Quiz Problem 2

- Break \( Y(s) \) into partial fractions:

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Y(s) = \frac{s}{(s-2)(s-1)^2} = \frac{A}{(s-1)^2} + \frac{B}{s-1} + \frac{C}{s-2}
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\[
\frac{B}{s-1} = \frac{s}{(s-2)(s-1)^2} - \frac{-1}{(s-1)^2} - \frac{2}{s-2}
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\]

\[
= \frac{2 - 2(s-1)}{(s-2)(s-1)} = -\frac{2s + 4}{(s-2)(s-1)} = \frac{-2}{s-1}
\]
Quiz Problem 2

Break $Y(s)$ into partial fractions:

$$Y(s) = \frac{s}{(s - 2)(s - 1)^2} = \frac{A}{(s - 1)^2} + \frac{B}{s - 1} + \frac{C}{s - 2}$$

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$$\frac{B}{s - 1} = \frac{s}{(s - 2)(s - 1)^2} - \frac{-1}{(s - 1)^2} - \frac{2}{s - 2}$$

$$= \frac{s + (s - 2) - 2(s - 1)^2}{(s - 2)(s - 1)^2} = \frac{2(s - 1) - 2(s - 1)^2}{(s - 2)(s - 1)^2}$$

$$= \frac{2 - 2(s - 1)}{(s - 2)(s - 1)} = \frac{-2s + 4}{(s - 2)(s - 1)} = \frac{-2}{s - 1}$$

So $B = -2$
Now that

\[ Y(s) = -\frac{1}{(s - 1)^2} - \frac{2}{s - 1} + \frac{2}{s - 2}, \]
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we perform the inverse Laplace transform.
Quiz Problem 2

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\[ Y(s) = -\frac{1!}{(s - 1)^2} \]
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\[ Y(s) = -\frac{1}{(s-1)^2} - \frac{2}{s-1} + \frac{2}{s-2}, \]

we perform the inverse Laplace transform. Notice that

\[ Y(s) = -\frac{1!}{(s-1)^2} - 2\frac{1}{s-1} \]
Now that

\[ Y(s) = -\frac{1}{(s - 1)^2} - \frac{2}{s - 1} + \frac{2}{s - 2}, \]

we perform the inverse Laplace transform. Notice that

\[ Y(s) = -\frac{1!}{(s - 1)^2} - 2\frac{1}{s - 1} + 2\frac{1}{s - 2}, \]
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\[ Y(s) = -\frac{1!}{(s - 1)^2} - 2\frac{1}{s - 1} + 2\frac{1}{s - 2}, \]

from the formulas you are supposed to memorize,
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from the formulas you are supposed to memorize,
\[ y(t) = -e^t t^2 \]
Quiz Problem 2

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\[ y(t) = -e^t t^2 - 2e^t + 2e^{2t} \]
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we perform the inverse Laplace transform. Notice that
\[ Y(s) = -\frac{1!}{(s - 1)^2} - 2\frac{1}{s - 1} + 2\frac{1}{s - 2}, \]
from the formulas you are supposed to memorize,
\[ y(t) = -e^t t^2 - 2e^t + 2e^{2t} \]
and the ODE is solved.
Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]
Quiz Problem 1

Find the Laplace transform of the function

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- By integration by parts:
Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

- By integration by parts:

\[ I = \int_0^\infty e^{-st} e^t \cos 3t \, dt \]
Quiz Problem 1

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- By integration by parts:

\[
\begin{align*}
I &= \int_0^\infty e^{-st} e^t \cos 3t \, dt = \int_0^\infty e^{(1-s)t} \cos 3t \, dt
\end{align*}
\]
Quiz Problem 1

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\[
= \int_0^\infty e^{(1-s)t} \frac{1}{3} d \sin 3t
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\[ = \frac{1}{3} e^{(1-s)t} \sin 3t \bigg|_0^\infty - \frac{1}{3} \int_0^\infty \sin 3t de^{(1-s)t} \]
Quiz Problem 1

Find the Laplace transform of the function

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\[
= \frac{1}{3} e^{(1-s)t} \sin 3t \bigg|_0^\infty - \frac{1}{3} \int_0^\infty \sin 3t e^{(1-s)t} \, dt
\]

\[
= 0 - 0 - \frac{1-s}{3} \int_0^\infty e^{(1-s)t} \sin 3t \, dt
\]
Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

(continued)

\[
\begin{align*}
I &= 0 - 0 - \frac{1-s}{3} \int_0^\infty e^{(1-s)t} \sin 3tdt \\
\end{align*}
\]
Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

(continued)

\[
I = 0 - 0 - \frac{1 - s}{3} \int_0^\infty e^{(1-s)t} \sin 3tdt
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Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

(continued)

\[ I = 0 - 0 - \frac{1-s}{3} \int_0^\infty e^{(1-s)t} \sin 3t \, dt \]

\[ = \frac{1-s}{3} \int_0^\infty e^{(1-s)t} \frac{1}{3} d \cos 3t \]

\[ = \frac{1-s}{9} e^{(1-s)t} \cos 3t \bigg|_0^\infty - \frac{1-s}{9} \int_0^\infty \cos 3t \, de^{(1-s)t} \]
Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

(continued)

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\[ = \frac{1-s}{9} e^{(1-s)t} \cos 3t \bigg|_0^\infty - \frac{1-s}{9} \int_0^\infty \cos 3t d e^{(1-s)t} \]

\[ = 0 - \frac{1-s}{9} - \frac{(1-s)^2}{9} \int_0^\infty e^{(1-s)t} \cos 3tdt = \frac{s-1}{9} - \frac{(1-s)^2}{9} I \]
Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

(continued)

\[ I = 0 - 0 - \frac{1-s}{3} \int_0^\infty e^{(1-s)t} \sin 3tdt \]

\[ = \frac{1-s}{3} \int_0^\infty e^{(1-s)t} \frac{1}{3} d\cos 3t \]

\[ = \frac{1-s}{9} e^{(1-s)t} \cos 3t \bigg|_0^\infty - \frac{1-s}{9} \int_0^\infty \cos 3t d(e^{(1-s)t}) \]

\[ = 0 - \frac{1-s}{9} - \frac{(1-s)^2}{9} \int_0^\infty e^{(1-s)t} \cos 3tdt = \frac{s-1}{9} - \frac{(1-s)^2}{9} I \]

Therefore

\[ I = \frac{s-1}{9} - \frac{(1-s)^2}{9} \]

Therefore

\[ I = \frac{s-1}{9} \frac{1}{1 + \frac{(1-s)^2}{9}} \]
Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

(continued)

\[
I = 0 - 0 - \frac{1 - s}{3} \int_0^\infty e^{(1-s)t} \sin 3tdt
\]

\[
= \frac{1 - s}{3} \int_0^\infty e^{(1-s)t} \frac{1}{3} d \cos 3t
\]

\[
= \frac{1 - s}{9} e^{(1-s)t} \cos 3t \bigg|_0^\infty - \frac{1 - s}{9} \int_0^\infty \cos 3t d e^{(1-s)t}
\]

\[
= 0 - \frac{1 - s}{9} - \frac{(1 - s)^2}{9} \int_0^\infty e^{(1-s)t} \cos 3tdt = \frac{s - 1}{9} - \frac{(1 - s)^2}{9} I
\]

Therefore

\[
I = \frac{\frac{s-1}{9}}{1 + \frac{(1-s)^2}{9}} = \frac{s - 1}{(s - 1)^2 + 9}
\]
Quiz Problem 1

Find the Laplace transform of the function

\[ f(t) = e^t \cos 3t. \]

- By exponential shift formula:
Quiz Problem 1

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Putting \( f(t) = \cos 3t \) into the formula
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Quiz Problem 1

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- By complexification: we just need to figure the real part of \( \mathcal{L}(e^t e^{3it}) \).
Quiz Problem 1

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\[ \mathcal{L}(e^t e^{3it}) = \mathcal{L}(e^{(1+3i)t}) = \frac{1}{s - 1 - 3i}, \]
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\[ \mathcal{L}(e^t e^{3it}) = \frac{s - 1 + 3i}{(s - 1)^2 + 9}. \]
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Find the Laplace transform of the function

$$f(t) = e^t \cos 3t.$$

- By complexification: we just need to figure the real part of $\mathcal{L}(e^t e^{3it})$. Since

$$\mathcal{L}(e^t e^{3it}) = \mathcal{L}(e^{(1+3i)t}) = \frac{1}{s - 1 - 3i},$$

by arithmetic of complex numbers (that multiplies the conjugate of the denominator both at top and at bottom):

$$\mathcal{L}(e^t e^{3it}) = \frac{s - 1 + 3i}{(s - 1)^2 + 9} = \frac{s - 1}{(s - 1)^2 + 9} + i \frac{3}{(s - 1)^2 + 9},$$
Quiz Problem 1

Find the Laplace transform of the function

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Since

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then

\[ \mathcal{L}(e^t \cos 3t) = \text{the real part of} \mathcal{L}(e^t e^{3it}) = \frac{s - 1}{(s - 1)^2 + 9}. \]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \ y'(0) = 1, \ y''(0) = 0, \ y'''(0) = 1 \]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

Perform the Laplace transform:

\[
\begin{align*}
\mathcal{L}\{y(t)\} &= \frac{y(0)}{s} + \frac{y'(0)}{s^2} + \frac{y''(0)}{s^3} + \frac{y'''(0)}{s^4} + \mathcal{L}\{y^{(4)}\}(s),
\end{align*}
\]

\[
\begin{align*}
\mathcal{L}\{y(t)\} &= \frac{4}{s^3} - \frac{6}{s^2} + \frac{1}{s} + \mathcal{L}\{y^{(4)}\}(s),
\end{align*}
\]

By algebra one gets

\[ Y(s) = \frac{s^2 + 7}{s^4 + 4s^3 + 7s^2 + 4s + 1} \]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Perform the Laplace transform: Let \( Y(s) = \mathcal{L}(y(t)) \).
Solve the IVP

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\[
s^4 Y(s) - s^3 y(0) - s^2 y'(0) - sy''(0) - y'''(0)
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Homework Problem 6.2.17

Solve the IVP

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\[
s^4 Y(s) - s^3 y(0) - s^2 y'(0) - sy''(0) - y'''(0) = s^4 Y(s) - s^2 - 1,
\]

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Homework Problem 6.2.17

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\[
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s^3 Y(s) - s^2 y(0) - sy'(0) - y''(0) &= s^3 Y(s) - s^2 - 1,
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\[
\begin{align*}
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    s^3 Y(s) - s^2 y(0) - sy'(0) - y''(0) &= s^3 Y(s) - s,
\end{align*}
\]
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Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Perform the Laplace transform: Let \( Y(s) = \mathcal{L}(y(t)) \). By

\[
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s^3Y(s) - s^2y(0) - sy'(0) - y''(0) &= s^3 Y(s) - s, \\
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\[
\begin{align*}
   s^4 Y(s) - s^3 y(0) - s^2 y'(0) - sy''(0) - y''(0) &= s^4 Y(s) - s^2 - 1, \\
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   s^2 Y(s) - sy(0) - y'(0) &= s^2 Y(s) - 1, \\
   sY(s) - y(0) &= sY(s) - y(0)
\end{align*}
\]
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\[
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    s^2 Y(s) - sy(0) - y'(0) &= s^2 Y(s) - 1, \\
    sY(s) - y(0) &= sY(s),
\end{align*}
\]

the original ODE becomes
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Perform the Laplace transform: Let \( Y(s) = L(y(t)) \). By
  
  \[
  s^4 Y(s) - s^3 y(0) - s^2 y'(0) - s y''(0) - y'''(0) = s^4 Y(s) - s^2 - 1,
  
  s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) = s^3 Y(s) - s,
  
  s^2 Y(s) - s y(0) - y'(0) = s^2 Y(s) - 1, \quad sY(s) - y(0) = sY(s),
  
  the original ODE becomes
  
  \[
  (s^4 - 4s^3 + 6s^2 - 4s + 1)Y(s)
  \]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

Perform the Laplace transform: Let \( Y(s) = \mathcal{L}(y(t)) \). By

\[
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    s^4 Y(s) - s^3 y(0) - s^2 y'(0) - s y''(0) - y'''(0) & = s^4 Y(s) - s^2 - 1, \\
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    s^2 Y(s) - s y(0) - y'(0) & = s^2 Y(s) - 1, \\
    sY(s) - y(0) & = sY(s),
\end{align*}
\]

the original ODE becomes

\[
(s^4 - 4s^3 + 6s^2 - 4s + 1) Y(s) - s^2 - 1
\]
Homework Problem 6.2.17

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\end{align*}
\]

the original ODE becomes

\[
(s^4 - 4s^3 + 6s^2 - 4s + 1)Y(s) - s^2 - 1 + 4s
\]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

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\end{align*}
\]

the original ODE becomes

\[
(s^4 - 4s^3 + 6s^2 - 4s + 1)Y(s) - s^2 - 1 + 4s - 6\]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Perform the Laplace transform: Let \( Y(s) = \mathcal{L}(y(t)) \). By

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  s^2 Y(s) - sy(0) - y'(0) &= s^2 Y(s) - 1, \\
  sY(s) - y(0) &= sY(s),
\end{align*}
\]

the original ODE becomes

\[
(s^4 - 4s^3 + 6s^2 - 4s + 1)Y(s) - s^2 - 1 + 4s - 6 = 0
\]

By algebra one gets
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

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\[
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    s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) &= s^3 Y(s) - s, \\
    s^2 Y(s) - s y(0) - y'(0) &= s^2 Y(s) - 1, \\
    s Y(s) - y(0) &= s Y(s),
\end{align*}
\]

the original ODE becomes

\[ (s^4 - 4s^3 + 6s^2 - 4s + 1) Y(s) - s^2 - 1 + 4s - 6 = 0 \]

By algebra one gets

\[ Y(s) = \frac{s^2 - 4s + 7}{(s - 1)^4} \]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Break \( Y(s) \) into partial fractions.
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Break \( Y(s) \) into partial fractions. Let’s use cover up method here.
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Break \( Y(s) \) into partial fractions. Let’s use cover up method here. Let

\[
\frac{s^2 - 4s + 7}{(s - 1)^4}
\]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Break \( Y(s) \) into partial fractions. Let’s use cover up method here. Let

\[
\frac{s^2 - 4s + 7}{(s - 1)^4} = \frac{A}{(s - 1)^4} + \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Break \( Y(s) \) into partial fractions. Let’s use cover up method here. Let

\[
\frac{s^2 - 4s + 7}{(s - 1)^4} = \frac{A}{(s - 1)^4} + \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

The cover up method gives
Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

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\]

The cover up method gives

\[ A = (s^2 - 4s + 7)|_{s=1} \]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

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The cover up method gives

\[ A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 \]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Break \( Y(s) \) into partial fractions. Let’s use cover up method here. Let

\[
\frac{s^2 - 4s + 7}{(s - 1)^4} = \frac{A}{(s - 1)^4} + \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

The cover up method gives

\[
A = \left. (s^2 - 4s + 7) \right|_{s=1} = 1 - 4 + 7 = 4
\]
Homework Problem 6.2.17

Solve the IVP
\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

- Break \( Y(s) \) into partial fractions. Let’s use cover up method here. Let

\[
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\]

The cover up method gives

\[
A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4
\]

Subtract the left-hand-side with \( 4/(s - 1)^4 \),
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

Break \( Y(s) \) into partial fractions. Let’s use cover up method here. Let

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\frac{s^2 - 4s + 7}{(s - 1)^4} = \frac{A}{(s - 1)^4} + \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

The cover up method gives

\[ A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4 \]

Subtract the left-hand-side with \( 4/(s - 1)^4 \), one gets

\[
\frac{s^2 - 4s + 7 - 4}{(s - 1)^4}
\]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

Break \( Y(s) \) into partial fractions. Let’s use cover up method here. Let

\[
\frac{s^2 - 4s + 7}{(s - 1)^4} = \frac{A}{(s - 1)^4} + \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

The cover up method gives

\[ A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4 \]

Subtract the left-hand-side with \( 4/(s - 1)^4 \), one gets

\[
\frac{s^2 - 4s + 7 - 4}{(s - 1)^4} = \frac{s - 3}{(s - 1)^3}
\]
Homework Problem 6.2.17

Solve the IVP
\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

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\]

The cover up method gives

\[ A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4 \]

Subtract the left-hand-side with \( 4/(s - 1)^4 \), one gets

\[
\frac{s^2 - 4s + 7 - 4}{(s - 1)^4} = \frac{s - 3}{(s - 1)^3} = \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

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The cover up method gives

\[ A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4 \]

Subtract the left-hand-side with \( 4/(s - 1)^4 \), one gets

\[
\frac{s^2 - 4s + 7 - 4}{(s - 1)^4} = \frac{s - 3}{(s - 1)^3} = \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Cover-up method gives
Homework Problem 6.2.17

Solve the IVP

\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \ y(0) = 0, \ y'(0) = 1, \ y''(0) = 0, \ y'''(0) = 1 \]

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\]

The cover up method gives

\[ A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4 \]

Subtract the left-hand-side with \( 4/(s - 1)^4 \), one gets

\[
\frac{s^2 - 4s + 7 - 4}{(s - 1)^4} = \frac{s - 3}{(s - 1)^3} = \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Cover-up method gives

\[ B = (s - 3)|_{s=1} \]
Homework Problem 6.2.17

Solve the IVP
\[ y^{(4)} - 4y^{(3)} + 6y'' - 4y' + 1 = 0, \quad y(0) = 0, \quad y'(0) = 1, \quad y''(0) = 0, \quad y'''(0) = 1 \]

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\]

The cover up method gives
\[ A = (s^2 - 4s + 7)|_{s=1} = 1 - 4 + 7 = 4 \]

Subtract the left-hand-side with \( 4/(s - 1)^4 \), one gets
\[
\frac{s^2 - 4s + 7 - 4}{(s - 1)^4} = \frac{s - 3}{(s - 1)^3} = \frac{B}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Cover-up method gives
\[ B = (s - 3)|_{s=1} = -2. \]
Homework Problem 6.2.17

Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3) + 2}{(s - 1)^3} = \frac{1}{(s - 1)^2}$$

Then immediately $C = 1$ and $D = 0$.

So

$$Y(s) = 4\frac{(s - 1)^4}{(s - 1)^3} + \frac{1}{(s - 1)^2}$$

Perform the inverse transformation. The formula one should use here is the exponential-shift formula.

Since

$$Y(s) = 4\frac{3!}{3!} \frac{3!}{3!} + \frac{1}{2!} + \frac{1}{1!}$$

$$y(t) = 4\frac{6}{3}e^{t}t^{3} + \frac{1}{2}e^{t}t^{2} + e^{t}t$$

Fei Qi (Rutgers University)
Break $Y(s)$ into partial fractions (continued): So

\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Again by subtraction one gets
Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3 + 2)}{(s - 1)^3}$$
Break \( Y(s) \) into partial fractions (continued):

\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Again by subtraction one gets

\[
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2}
\]
Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

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Break $Y(s)$ into partial fractions (continued): So

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\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
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Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Then immediately $C = 1$ and $D = 0$. 

Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$Y(s) = \frac{4}{3!} s^{4 - 2} + \frac{1}{2!} s^{2 - 2} = 2 \frac{3}{2} t^3 e^{t} + te^{t}.$$
Homework Problem 6.2.17

- Break \( Y(s) \) into partial fractions (continued):

\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Again by subtraction one gets

\[
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Then immediately \( C = 1 \) and \( D = 0 \). So

\[
Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
\]
Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

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$$Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.$$
Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So
  \[
  \frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
  \]
  Again by subtraction one gets
  \[
  \frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
  \]
  Then immediately $C = 1$ and $D = 0$. So
  \[
  Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
  \]

- Perform the inverse transformation. The formula one should use here is the exponential-shift formula.
Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Then immediately $C = 1$ and $D = 0$. So

$$Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.$$ 

- Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$Y(s)$$
Homework Problem 6.2.17

- Break \( Y(s) \) into partial fractions (continued): So

\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
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Again by subtraction one gets

\[
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Then immediately \( C = 1 \) and \( D = 0 \). So

\[
Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
\]

- Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

\[
Y(s) = \frac{4}{3!} \frac{3!}{(s - 1)^4}
\]
Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Then immediately $C = 1$ and $D = 0$. So

$$Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.$$ 

Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$Y(s) = \frac{4}{3!} \frac{3!}{(s - 1)^4} - \frac{2}{(s - 1)^3}$$
Break $Y(s)$ into partial fractions (continued): So

\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Again by subtraction one gets

\[
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Then immediately $C = 1$ and $D = 0$. So

\[
Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
\]

Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

\[
Y(s) = \frac{4}{3!} \frac{3!}{(s - 1)^4} - \frac{2!}{(s - 1)^3} + \frac{1!}{(s - 1)^2}
\]
Break $Y(s)$ into partial fractions (continued): So

\[ \frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1} \]

Again by subtraction one gets

\[ \frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1} \]

Then immediately $C = 1$ and $D = 0$. So

\[ Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}. \]

Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

\[ Y(s) = \frac{4}{3!} \frac{3!}{(s - 1)^4} - \frac{2}{2!} \frac{2!}{(s - 1)^3} + \frac{1}{1!} \frac{1!}{(s - 1)^2} \]

\[ y(t) = \frac{4}{3!} (s - 1)^4 - \frac{2}{2!} (s - 1)^3 + \frac{1}{1!} (s - 1)^2. \]
Break \( Y(s) \) into partial fractions (continued): So

\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Again by subtraction one gets

\[
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Then immediately \( C = 1 \) and \( D = 0 \). So

\[
Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
\]

Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

\[
Y(s) = \frac{4}{3! (s - 1)^4} - \frac{2}{2! (s - 1)^3} + \frac{1}{1! (s - 1)^2}
\]

\[
y(t) = \frac{4}{3!} e^t t^3
\]
Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Then immediately $C = 1$ and $D = 0$. So

$$Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.$$  

- Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$Y(s) = \frac{4}{3!} \frac{3!}{(s - 1)^4} - \frac{2!}{(s - 1)^3} + \frac{1!}{(s - 1)^2}$$

$$y(t) = 4 e^t t^3 - e^t t^2$$
Break $Y(s)$ into partial fractions (continued): So

$$\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Again by subtraction one gets

$$\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}$$

Then immediately $C = 1$ and $D = 0$. So

$$Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}. $$

Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$Y(s) = \frac{4}{3! (s - 1)^4} - \frac{2}{2! (s - 1)^3} + \frac{1}{1! (s - 1)^2}$$

$$y(t) = \frac{4}{3!} e^t t^3 - \frac{2}{2!} e^t t^2 + e^t t$$
Break $Y(s)$ into partial fractions (continued): So
\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]
Again by subtraction one gets
\[
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]
Then immediately $C = 1$ and $D = 0$. So
\[
Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
\]
Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since
\[
Y(s) = \frac{4}{3!} \frac{3!}{(s - 1)^4} - \frac{2!}{(s - 1)^3} + \frac{1!}{(s - 1)^2}
\]
\[
y(t) = \frac{4}{3} e^t t^3 - e^t t^2 + e^t t = \frac{2}{3} t^3 e^t.
\]
Break $Y(s)$ into partial fractions (continued): So

$$
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
$$

Again by subtraction one gets

$$
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
$$

Then immediately $C = 1$ and $D = 0$. So

$$
Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
$$

Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

$$
Y(s) = \frac{4}{3! (s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}
$$

$$
y(t) = \frac{4}{3!} e^t t^3 - e^t t^2 + e^t t = \frac{2}{3} t^3 e^t - t^2 e^t
$$
Homework Problem 6.2.17

- Break $Y(s)$ into partial fractions (continued): So

\[
\frac{s - 3}{(s - 1)^3} = \frac{-2}{(s - 1)^3} + \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Again by subtraction one gets

\[
\frac{(s - 3 + 2)}{(s - 1)^3} = \frac{1}{(s - 1)^2} = \frac{C}{(s - 1)^2} + \frac{D}{s - 1}
\]

Then immediately $C = 1$ and $D = 0$. So

\[
Y(s) = \frac{4}{(s - 1)^4} - \frac{2}{(s - 1)^3} + \frac{1}{(s - 1)^2}.
\]

- Perform the inverse transformation. The formula one should use here is the exponential-shift formula. Since

\[
Y(s) = \frac{4}{3!} \frac{3!}{(s - 1)^4} - \frac{2!}{(s - 1)^3} + \frac{1!}{(s - 1)^2}
\]

\[
y(t) = \frac{4}{3} e^t t^3 - e^t t^2 + e^t = \frac{2}{3} t^3 e^t - t^2 e^t + t e^t.
\]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0 \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases} , \quad y(0) = 0, \ y'(0) = 0 \]

- Express the right hand side in a single closed formula.
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases} , 
 y(0) = 0, y'(0) = 0 \]

Express the right hand side in a single closed formula. By what you have learned in 6.3,
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, \, y'(0) = 0 \]

- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, \quad y(0) = 0, \quad y'(0) = 0 \]

Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

\[ y'' + y = t - t u_1(t). \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
    t & 0 \leq t < 1 \\
    0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, y'(0) = 0 \]

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- Perform the Laplace transform
Homework Problem 6.2.25

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- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

  \[ y'' + y = t - tu_1(t). \]

- Perform the Laplace transform using the derivative formula and \( t \)-axis translation formula:
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases} , \quad y(0) = 0, y'(0) = 0 \]

- Express the right hand side in a single closed formula. By what you have learned in 6.3, the ODE can be written as

  \[ y'' + y = t - tu_1(t). \]

- Perform the Laplace transform using the derivative formula and t-axis translation formula:

  \[ s^2Y(s) + Y(s) = \frac{1}{s^2} \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
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- Perform the Laplace transform using the derivative formula and \( t \)-axis translation formula:
  \[ s^2Y(s) + Y(s) = \frac{1}{s^2} - e^{-s} \mathcal{L}(t + 1) \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
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\[
\begin{align*}
  s^2 Y(s) + Y(s) &= \frac{1}{s^2} - e^{-s} \mathcal{L}(t + 1) \\
  &= \frac{1}{s^2}
\end{align*}
\]
Homework Problem 6.2.25

Solve the IVP

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   s^2 Y(s) + Y(s) &= \frac{1}{s^2} - e^{-s} \mathcal{L}(t + 1) \\
                     &= \frac{1}{s^2} - e^{-s} \mathcal{L}(t) - e^{-s} \mathcal{L}(1)
\end{align*}
\]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
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  \[ = \frac{1}{s^2} - e^{-s} \mathcal{L}(t) - e^{-s} \mathcal{L}(1) \]
  \[ = \frac{1}{s^2} \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
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  s^2 Y(s) + Y(s) &= \frac{1}{s^2} - e^{-s} \mathcal{L}(t + 1) \\
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  &= \frac{1}{s^2} - e^{-s} \frac{1}{s^2} 
\end{align*}
\]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
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  \[ = \frac{1}{s^2} - e^{-s}\mathcal{L}(t) - e^{-s}\mathcal{L}(1) \]

  \[ = \frac{1}{s^2} - e^{-s} \frac{1}{s^2} - e^{-s} \frac{1}{s} \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, \ y'(0) = 0 \]

- Perform the Laplace transform (continued):
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, \quad y(0) = 0, y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{s^2 + 1} \]
Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, \ y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, \quad y(0) = 0, \; y'(0) = 0 \]

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- Find the inverse Laplace transform.
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
    t & 0 \leq t < 1 \\
    0 & 1 \leq t < \infty
\end{cases}, \quad y(0) = 0, \ y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]

- Find the inverse Laplace transform. By whatever method you have,
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases} \ , \ y(0) = 0, \ y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[
Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}.
\]

- Find the inverse Laplace transform. By whatever method you have,

\[
\frac{1}{(s^2 + 1)s^2}
\]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases} , \quad y(0) = 0, \quad y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s} - \frac{e^{-s}}{(s^2 + 1)s} . \]

- Find the inverse Laplace transform. By whatever method you have,

\[ \frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \]
Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, \; y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]

- Find the inverse Laplace transform. By whatever method you have,

\[ \frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \quad \frac{1}{s(s^2 + 1)} \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]

- Find the inverse Laplace transform. By whatever method you have,

\[ \frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \quad \frac{1}{s} = 1 - \frac{s}{s^2 + 1}. \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, \quad y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]

- Find the inverse Laplace transform. By whatever method you have,

\[ \frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \quad \frac{1}{s(s^2 + 1)} = \frac{1}{s} - \frac{s}{s^2 + 1} \]

Then

\[ y(t) = t \sin t + u_1(t) \left[ t \sin(1) + 1 \cos(1) \right] \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} \ t & 0 \leq t < 1 \\ \ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]

- Find the inverse Laplace transform. By whatever method you have,

\[
\frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \quad \frac{1}{s(s^2 + 1)} = \frac{1}{s} - \frac{s}{s^2 + 1}.
\]

Then

\[ y(t) = (t - \sin t) \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, \quad y(0) = 0, \ y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

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Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. 
\]

- Find the inverse Laplace transform. By whatever method you have,

\[
\frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \quad \frac{1}{s(s^2 + 1)} = \frac{1}{s} - \frac{s}{s^2 + 1}
\]

Then

\[ y(t) = (t - \sin t) + u_1(t) \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} t & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, \quad y(0) = 0, y'(0) = 0 \]

Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s} . \]

Find the inverse Laplace transform. By whatever method you have,

\[ \frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \quad \frac{1}{s(s^2 + 1)} = \frac{1}{s} - \frac{s}{s^2 + 1} \]

Then

\[ y(t) = (t - \sin t) + u_1(t) [t - 1] \]
Solve the IVP

\[ y'' + y = \begin{cases} t & \text{if } 0 \leq t < 1 \\ 0 & \text{if } 1 \leq t < \infty \end{cases} , \quad y(0) = 0, \quad y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]

- Find the inverse Laplace transform. By whatever method you have,

\[ \frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1} \quad \frac{1}{s(s^2 + 1)} = \frac{1}{s} - \frac{s}{s^2 + 1} \]

Then

\[ y(t) = (t - \sin t) + u_1(t) [t - 1 - \sin(t - 1)] \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} \frac{t}{s^2 + 1} & 0 \leq t < 1 \\ 0 & 1 \leq t < \infty \end{cases}, y(0) = 0, y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

\[ Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}. \]

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Then

\[ y(t) = (t - \sin t) + u_1(t) [t - 1 - \sin(t - 1) + 1] \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
  t & 0 \leq t < 1 \\
  0 & 1 \leq t < \infty 
\end{cases}, \quad y(0) = 0, \quad y'(0) = 0 \]

- Perform the Laplace transform (continued): So after algebra,

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Then

\[ y(t) = (t - \sin t) + u_1(t) [t - 1 - \sin(t - 1) + 1 - \cos(t - 1)] \]
Homework Problem 6.2.25

Solve the IVP

\[ y'' + y = \begin{cases} 
    t & 0 \leq t < 1 \\
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Y(s) = \frac{1}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s^2} - \frac{e^{-s}}{(s^2 + 1)s}.
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- Find the inverse Laplace transform. By whatever method you have,

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\frac{1}{(s^2 + 1)s^2} = \frac{1}{s^2} - \frac{1}{s^2 + 1}, \quad \frac{1}{s(s^2 + 1)} = \frac{1}{s} - \frac{s}{s^2 + 1}
\]

Then

\[
y(t) = (t - \sin t) + u_1(t) [t - 1 - \sin(t - 1) + 1 - \cos(t - 1)]
\]
The End