What you should learn from Recitation 7:
a. Finding Particular Solutions of Inhomogenous ODE

Fei Qi

Rutgers University

fq15@math.rutgers.edu

March 13, 2014
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There may be errors. Use them at your own discretion. Anyone who notify me with an error will get some award in grade points.
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For the second order linear inhomogeneous ODE

\[ ay'' + by' + cy = g(t), \]
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If the \( g(t) \) is special enough, then you can proceed by guessing the solution. Otherwise, you have to use variation of parameters, which works for all cases but usually is less convenient.
How to Guess a particular solution

The template for your first try is summarized as below:

If \( g(t) \) looks like \( g(t) = e^{at}(an + an-1t + \ldots + a_0) \);
then try \( P(t) = e^{at}(A_n + A_{n-1}t + \ldots + A_0) \);
where \( A_n, A_0 \) are coefficients to be determined.

If \( g(t) \) looks like \( g(t) = e^{t}(a \cos t + b \sin t) \);
then try \( P(t) = e^{t}(A \cos t + B \sin t) \)
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If your first try fails, multiply your template with \( t \) and try again.
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The template for your first try is summarized as below:

- If $g(t)$ looks like

$$g(t) = e^{\alpha t}(a_n t^n + a_{n-1} t^{n-1} + \cdots + a_0),$$

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Examples and remarks

- \( g(t) = 3. \)
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- $g(t) = 3$. This case you have $\alpha = 0$. 

CAUTION: Even you have only 1 term, since your polynomial is of degree 2, THERE SHOULD BE 3 UNDETERMINED COEFFICIENTS. In general if your polynomial is of degree $n$, there should be $n + 1$ of undetermined coefficients, REGARDLESS how many the lower terms there are.
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Examples and remarks

- $g(t) = 3$. This case you have $\alpha = 0$ and $n = 0$. Try the template $P(t) = A$. 

- $g(t) = t^2 e^3$. This case you have $\alpha = 3$ and $n = 2$. Try the template $P(t) = e^t (A_2 t^2 + A_1 t + A_0)$. 

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- \( g(t) = e^t(t + 2) \), i.e., \( \alpha = 1 \), \( n = 1 \), \( a_1 = 1 \), \( a_0 = 2 \). Try the template \( P(t) = e^t(A_1t + A_0) \).

- \( g(t) = t^2e^{3t} \), i.e., \( \alpha = 3 \), \( n = 2 \), \( a_2 = 1 \), \( a_1 = a_0 = 0 \). Try the template \( P(t) = e^t(A_2t^2 + A_1t + A_0) \).

**CAUTION:** Even you have only 1 term, since your polynomial is of degree 2, THERE SHOULD BE 3 UNDETERMINED COEFFICIENTS. In general if your polynomial is of degree \( n \), there should be \( n + 1 \) of undetermined coefficients, REGARDLESS how many the lower terms there are.
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- \( g(t) = e^t t^2 \cos t \). This case is in fact too complicated to use this method. Instead, use variation of parameters.
Facts that will be used in solving problems

- General fact:

\[
\text{If } P_1(t) \text{ is a particular solution for } ay'' + by' + cy = g_1(t), \text{ and } P_2(t) \text{ is a particular solution for } ay'' + by' + cy = g_2(t), \text{ then } P_1(t) + P_2(t) \text{ is a particular solution for } ay'' + by' + cy = g_1(t) + g_2(t). \]

Exercise: Prove this fact. (very easy) So for example, if the ODE looks like

\[
y'' + 2y' + 3y = e^t(t^2 + 4) + e^{-3t}\cos 3t + \cos 4t + t^2.\]

Then

\[
P(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t)
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So for example, if the ODE looks like $y'' + 2y' + 3y = e^t(t^2 + 4) + e^{-3t} \cos 3t + \cos 4t + t^2$; then you should find particular solutions $P_1(t)$ for $y'' + 2y' + 3y = e^t(t^2 + 4)$, $P_2(t)$ for $y'' + 2y' + 3y = e^{-3t} \cos 3t$, $P_3(t)$ for $y'' + 2y' + 3y = \cos 4t$, and $P_4(t)$ for $y'' + 2y' + 3y = t^2$. Then $P(t) = P_1(t) + P_2(t) + P_3(t) + P_4(t)$ is a particular solution for this ODE.
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However in industry, $g(t)$ may have 40 to 50 summands and that’s why you should learn how to use mathematica / maple.
Exponential-Shift Rule

In computations, dealing with multiple derivatives often leads to messy algebras.

Theorem

Let $f(x)$ be a polynomial, denote $D = \frac{d}{dt}$, the derivative operator. Then for any function $u(t)$ and for any number $a$,

$$f(D)(e^{at}u(t)) = e^{at}(f(D+a)u(t)).$$

In words, to move $e^{at}$ out of $f(D)(e^{at}u(t))$, you should pay the price of modifying $f(D)$ into $f(D+a)$, getting $e^{at}(f(D+a)u(t))$. 
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Fei Qi (Rutgers University)  Recitation 7: a. Finding particular solutions  March 13, 2014  10 / 42
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In words, to move \( e^{at} \) out of \( f(D)(e^{at}u(t)) \), you should pay the price of modifying \( f(D) \) into \( f(D + a) \), getting \( e^{at}(f(D + a)u(t)) \).
Exponential-Shift Rule

Remarks:

Please check 27:00 of the MIT Lecture 13 for a proof. (Note that he used a different symbol $p$)

Please check 0:00 of the MIT Lecture 14 for the interpretation of $f(D)$.

In our scenario where the ODE is $ay'' + by' + cy = g(t)$, basically we will always take $f(x) = ax^2 + bx + c$.

The most cited example of $f(D)$ acting on a function is $f(D)y = (a \frac{d}{dt}^2 + b \frac{d}{dt} + c)y = ay'' + by' + cy$:

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Exponential-Shift Rule

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### Fei Qi (Rutgers University)

Recitation 7: a. Finding particular solutions

March 13, 2014

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Exponential-Shift Rule

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\[ f(D)y \]
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$$f(D)y = \left( a \frac{d^2}{dt^2} + b \frac{d}{dt} + c \right) y = ay'' + by' + cy.$$  

- You will see how this technique is used in the following example problems.
Find the general solution of

\[ y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t} \cos 3t + \cos 4t + t^2. \]
The ridiculously fabricated example problem

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- To get a particular solution, let’s get the particular solution for the 4 summands on the right hand side separately. Let’s first deal with the ODE

\[ y'' + 2y' - 3y = e^t(t^2 + 4). \]
The ridiculously fabricated example problem

The template you should try
The ridiculously fabricated example problem

- The template you should try is \( P(t) = e^t(At^2 + Bt + C) \)
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The template you should try is $P(t) = e^t(At^2 + Bt + C)$ (I hate writing subscripts so I’ll just use these three guys). In contrast to the way I taught in class, USE THE EXPONENTIAL-SHIFT RULE as follows.
Let $f(x) = x^2 + 2x - 3$
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Let $f(x) = x^2 + 2x - 3 = (x - 1)(x + 3)$. 
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Let \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \). Then

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$$= e^t f(D + 1)(At^2 + Bt + C)$$
$$= e^t(D + 1 - 1)(D + 1 + 3)(At^2 + Bt + C)$$
$$= e^t(D^2 + 4D)(At^2 + Bt + C).$$
The ridiculously fabricated example problem

- Notice that

\[
D \left( At^2 + Bt + C \right) = (2At + B)
\]

\[
D^2 \left( At^2 + Bt + C \right) = 2A + 4B
\]

Therefore

\[
P'' + 2P' + 3P = e^{t(8At + 2A + 4B)}
\]

But the right hand side we have is

\[
e^{t(t^2 + 4)}
\]

There is no term corresponds to \( t^2 e^t \) so our first try fails.

Remark: The first try always fails whenever your \( e^t \) corresponds to part of the complementary solutions (in this case \( e^t \)). And it is very easy to see this from the above arguments: the \( f(D + 1) \) above \( \text{DOES NOT HAVE A CONSTANT TERM!} \) Think about why this leads to the failure of first try.
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\end{align*}
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\[
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&= 2A + 4(2At + B) = 8At + 2A + 4B.
\end{align*}
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So

\[ (D^2 + 4D)(At^2 + Bt + C) \]
\[ = D^2(At^2 + Bt + C) + 4D(At^2 + Bt + C) \]
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\[ P'' + 2P' - 3P = e^t(8At + 2A + 4B). \]

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The ridiculously fabricated example problem

- Let’s do the second try:
  \[ P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct). \]
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\[ P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct). \]

Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \).
Let’s do the second try:

\[ P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct). \]

Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)).
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\[
P'' + 2P' - 3P = (D^2 + 2D - 3)P = f(D)P
\]
Let’s do the second try:

\[ P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct). \]
Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)):

\[
P'' + 2P' - 3P = (D^2 + 2D - 3)P = f(D)P
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Let’s do the second try:

\( P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct) \). Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)):

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P'' + 2P' - 3P = (D^2 + 2D - 3)P = f(D)P
= f(D)(e^t(At^3 + Bt^2 + Ct))
= e^t f(D + 1)(At^3 + Bt^2 + Ct)
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The ridiculously fabricated example problem

Let’s do the second try:
\[ P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct). \] Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)):

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= e^t(D^2 + 4D)(At^3 + Bt^2 + Ct).
\]
Let’s do the second try:

\[ P(t) = t e^t (At^2 + Bt + C) = e^t (At^3 + Bt^2 + Ct). \]

Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)):

\[
\begin{align*}
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
&= f(D)(e^t (At^3 + Bt^2 + Ct)) \\
&= e^t f(D + 1)(At^3 + Bt^2 + Ct) \\
&= e^t (D^2 + 4D)(At^3 + Bt^2 + Ct).
\end{align*}
\]

Notice that \( D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C \),
Let's do the second try:

\[ P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct). \]

Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)):

\[
P'' + 2P' - 3P &= (D^2 + 2D - 3)P = f(D)P \\
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Notice that

\[
D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C,
\]

\[
D^2(At^3 + Bt^2 + Ct) = 6At + 2B.
\]
Let’s do the second try:

\[ P(t) = te^t (At^2 + Bt + C) = e^t (At^3 + Bt^2 + Ct). \]

Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)):

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P'' + 2P' - 3P = (D^2 + 2D - 3)P = f(D)P
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\[
= f(D)(e^t (At^3 + Bt^2 + Ct))
\]

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= e^t f(D + 1)(At^3 + Bt^2 + Ct)
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Notice that

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\[ D^2(At^3 + Bt^2 + Ct) = 6At + 2B. \]

Then \( P'' + 2P' - 3P = \)

\[
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The ridiculously fabricated example problem

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\[ P(t) = te^t(At^2 + Bt + C) = e^t(At^3 + Bt^2 + Ct). \]

Again we use the exponential-shift rule to compute \( P'' + 2P' - 3P \) (Recall that \( f(x) = x^2 + 2x - 3 = (x - 1)(x + 3) \)):

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Notice that

\[
D(At^3 + Bt^2 + Ct) = 3At^2 + 2Bt + C, \\
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Then \( P'' + 2P' - 3P = e^t(D^2 + 4D)(At^3 + Bt^2 + Ct) \)
The ridiculously fabricated example problem

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The ridiculously fabricated example problem

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The ridiculously fabricated example problem

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\[ \Rightarrow P(t) = \left(\frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t\right)e^t. \]
The ridiculously fabricated example problem

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\[ = e^t(4\left(\frac{3}{12}t^2 - \frac{2}{16}t + \frac{33}{32}\right)) \]

So \( P(t) \) is a solution of \( y'' + 2y' - 3y = e^t(t^2 + 4) \) and the first part of the solution is done.
The ridiculously fabricated example problem

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The ridiculously fabricated example problem

Recall that our ODE is

\[ y'' + 2y' - 3y = e^t(t^2 + 4) + e^{-3t} \cos 3t + \cos 4t + t^2, \]

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  = e^{-3t}(D - 3 - 1)(D - 3 + 3)(A \cos 3t + B \sin 3t) \\
  = e^{-3t}(D^2 - 4D)(A \cos 3t + B \sin 3t).
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The ridiculously fabricated example problem

- Notice that

\[ D^2 (A \cos 3t + B \sin 3t) = 9A \cos 3t + 9B \sin 3t; \]

Then \((D^2 + 4D)(A \cos 3t + B \sin 3t) = (12A + 9B) \sin 3t + (9A + 12B) \cos 3t);\]

Thus \(P'' + 2P' = e^{-3t} (D^2 + 4D)(A \cos 3t + B \sin 3t) = e^{-3t} (12A + 9B) \sin 3t + (9A + 12B) \cos 3t);\]

Now compare the coefficients:

\(e^{-3t} (12A + 9B) \sin 3t + (9A + 12B) \cos 3t) = e^{-3t} \cos 3t) 12A + 9B = 0; 9A + 12B = 1;\]

\(A = 1 = 25; B = 4 = 75;\)
Notice that

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The ridiculously fabricated example problem

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\[(D^2 - 4D)(A \cos 3t + B \sin 3t)\]
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\[ D^2(A \cos 3t + B \sin 3t) = -9A \cos 3t - 9B \sin 3t. \]

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\[ (D^2 - 4D)(A \cos 3t + B \sin 3t) \]
\[ = -9A \cos 3t - 9B \sin 3t + 12A \sin 3t - 12B \cos 3t \]
The ridiculously fabricated example problem

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The ridiculously fabricated example problem

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The ridiculously fabricated example problem

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e^{-3t}[(12A - 9B) \sin 3t + (-9A - 12B) \cos 3t] = e^{-3t} \cos 3t
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The ridiculously fabricated example problem

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\[ (D^2 - 4D)(A \cos 3t + B \sin 3t) \]
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Now compare the coefficients:

\[ e^{-3t}[(12A - 9B) \sin 3t + (-9A - 12B) \cos 3t] = e^{-3t} \cos 3t \]
\[ \Rightarrow 12A - 9B = 0, \quad -9A - 12B = 1, \]
\[ \Rightarrow A = -1/25, \quad B = -4/75 \]
The ridiculously fabricated example problem

- So our particular solution would then be

\[ P(t) = e^{-3t} \left( \frac{1}{25} \cos 3t + \frac{4}{75} \sin 3t \right) : \text{CHECK!} \]

\[ P'' + 2P' + 3P = e^{-3t} \left( \frac{4}{25} (3 \sin 3t) + \frac{16}{75} (3 \cos 3t) + \frac{1}{25} (9 \cos 3t) + \frac{4}{75} 9 \right) = e^{-3t} (16 + \frac{9}{25} \cos 3t + \frac{12}{25} \frac{3}{25} \sin 3t) \]

So it is a particular solution.
The ridiculously fabricated example problem

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P'' + 2P' - 3P &= e^{-3t}(D^2 - 4D)(-\frac{1}{25}\cos 3t - \frac{4}{75}\sin 3t) \\
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&\quad + \frac{1}{25}(9\cos 3t) + \frac{4}{75}9\sin 35) \\
&= e^{-3t}\cos 3t.
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So it is a particular solution.
Remark:

If you have understood MIT Lecture 13 well enough, \( e^{-3t} \cos 3t \) corresponds to the characteristic root \( 3 + 3i \).

In other words, if the ODE looks like

\[
y'' + 6y' + 18 = e^{-3t} \cos 3t
\]

then your first try should fail.

Remark: From here you can also conclude that for second order linear homogeneous ODE, at most you need to make a second try. Think about why.
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The ridiculously fabricated example problem

Now let’s look at the third term:

\[ y'' + 2y' - 3y = \cos 4t \]
The ridiculously fabricated example problem

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The ridiculously fabricated example problem

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\[ P'' + 2P' - 3P = -16A\cos 4t - 16B\sin 4t \]
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P'' + 2P' - 3P & = -16A\cos 4t - 16B\sin 4t \\
+ & \quad 2(-4A\sin 4t + 4B\cos 4t)
\end{align*}
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Now let’s look at the third term:

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\]
\[
+ 2(-4A \sin 4t + 4B \cos 4t)
\]
\[
- 3(A \cos 4t + B \sin 4t)
\]

Comparing the coefficients, we have

\[
19A + 8B = 1
\]
\[
19B - 8A = 0
\]

Solving these equations gives

\[
A = \frac{19}{229}, \quad B = \frac{8}{229}
\]

Thus, the solution is

\[
P(t) = \frac{19}{229} \cos 4t + \frac{8}{229} \sin 4t
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= (-19A + 8B) \cos 4t + (-19B - 8A) \sin 4t.
\]

Compare the coefficients one has

\[-19A + 8B = 1,\]
The ridiculously fabricated example problem

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= (-19A + 8B)\cos 4t + (-19B - 8A)\sin 4t.
\]

- Compare the coefficients one has

\[-19A + 8B = 1, \quad -19B - 8A = 0, \]

\[\Rightarrow \quad A = -19B/8, \]

Fei Qi (Rutgers University)
The ridiculously fabricated example problem

Now let’s look at the third term:

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- The template is then \( P(t) = A \cos 4t + B \sin 4t \). Since you don’t have exponential here, it would be easy to compute directly:

\[
\begin{align*}
P'' + 2P' - 3P &= -16A \cos 4t - 16B \sin 4t \\
&\quad + 2(-4A \sin 4t + 4B \cos 4t) \\
&\quad - 3(A \cos 4t + B \sin 4t) \\
&= (-19A + 8B) \cos 4t + (-19B - 8A) \sin 4t.
\end{align*}
\]

- Compare the coefficients one has

\[-19A + 8B = 1, \quad -19B - 8A = 0,\]

\[\Rightarrow A = -19B/8, \quad (19^2 + 64)B = (361 + 64)B = 425B = 8,\]
The ridiculously fabricated example problem

Now let’s look at the third term:

\[ y'' + 2y' - 3y = \cos 4t \]

The template is then \( P(t) = A \cos 4t + B \sin 4t \). Since you don’t have exponential here, it would be easy to compute directly:

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\begin{align*}
P'' + 2P' - 3P &= -16A \cos 4t - 16B \sin 4t \\
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&- 3(A \cos 4t + B \sin 4t) \\
&= (-19A + 8B) \cos 4t + (-19B - 8A) \sin 4t.
\end{align*}
\]

Compare the coefficients one has

\[-19A + 8B = 1, \quad -19B - 8A = 0,\]

\[\Rightarrow A = -19B/8, \quad (19^2 + 64)B = (361 + 64)B = 425B = 8, \]

\[\Rightarrow A = -19/425,\]
Now let’s look at the third term:

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- Compare the coefficients one has

\[-19A + 8B = 1, \quad -19B - 8A = 0,\]

\[\Rightarrow A = -19B/8, \quad (19^2 + 64)B = (361 + 64)B = 425B = 8,\]

\[\Rightarrow A = -19/425, \quad B = 8/425.\]
So the particular solution is

\[ P(t) = -\frac{19}{425} \cos 4t + \frac{8}{425} \sin 4t. \]
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CHECK YOUR SOLUTION (skip).

Finally let’s look at the third term:

\[ y'' + 2y' - 3y = t^2 \]
The ridiculously fabricated example problem

The template is then

\[ P(t) = A t^2 + B t + C. \]

Again you don't have exponential and it would be easy to compute:

\[
\begin{align*}
P'' + 2P' + 3P &= 2A + 2(2At + B) \\
&= 2A + 4At + 2B \\
&= 2A + 4At + 2B \\
&= 2A + 4At + 2B \\
&= 2A + 4At + 2B \\
&= 2A + 2B \\
&= 2A + 2B \\
&= 2A + 2B.
\end{align*}
\]

Compare the coefficients one has

\[
\begin{align*}
3A &= 1; \\
4A + 3B &= 0; \\
2A + 2B + C &= 0.
\end{align*}
\]

\[
\begin{align*}
A &= \frac{1}{3}; \\
B &= \frac{1}{9}; \\
C &= (2A + 2B) = 3 = \frac{14}{3} = \frac{27}{9}.
\end{align*}
\]

\[
P(t) = \frac{1}{3} t^2 + \frac{1}{9} t + \frac{14}{27}.
\]

CHECK YOUR SOLUTION (skip).
The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. 

Again you don't have exponential and it would be easy to compute:

$$P'' + 2P' + 3P = 2A + 2(2At + B) + 3(At^2 + Bt + C) = 3At^2 + (4A + 3B)t + 2A + 2B + 3C$$

Compare the coefficients one has

- $3A = 1$;
- $4A + 3B = 0$;
- $2A + 2B + 3C = 0$.

Solving for $A$, $B$, and $C$:

- $A = 1/3$;
- $B = -4/9$;
- $C = (2A + 2B)/3 = 1/14 = 1/27$.

$P(t) = \frac{1}{3}t^2 - \frac{4}{9}t + \frac{1}{27}$. 

CHECK YOUR SOLUTION (skip).
The ridiculously fabricated example problem

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$$P'' + 2P' - 3P$$
*The ridiculously fabricated example problem*

- The template is then $P(t) = At^2 + Bt + C$. Again you don’t have exponential and it would be easy to compute:

\[ P'' + 2P' - 3P = 2A \]
The ridiculously fabricated example problem

The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$P'' + 2P' - 3P = 2A + 2(2At + B)$$
The ridiculously fabricated example problem

- The template is then $P(t) = A t^2 + B t + C$. Again you don’t have exponential and it would be easy to compute:

$$P'' + 2P' - 3P = 2A + 2(2At + B) - 3(At^2 + Bt + C)$$
The ridiculous example problem

The template is then $P(t) = At^2 + Bt + C$. Again you don’t have exponential and it would be easy to compute:

$$P'' + 2P' - 3P = 2A + 2(2At + B) - 3(At^2 + Bt + C)$$
$$= -3At^2 + (4A - 3B)t + 2A + 2B - 3C$$
The template is then $P(t) = At^2 + Bt + C$. Again you don’t have exponential and it would be easy to compute:

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Compare the coefficients one has
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\[-3A = 1,\]
The template is then \( P(t) = At^2 + Bt + C \). Again you don’t have exponential and it would be easy to compute:

\[
\begin{align*}
P'' + 2P' - 3P &= 2A + 2(2At + B) - 3(At^2 + Bt + C) \\
&= -3At^2 + (4A - 3B)t + 2A + 2B - 3C
\end{align*}
\]

Compare the coefficients one has

\[-3A = 1, 4A - 3B = 0,\]
The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

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$$-3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0$$

$\Rightarrow$ $A = -1/3,$
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Compare the coefficients one has

\[-3A = 1, 4A - 3B = 0, 2A + 2B - 3C = 0\]

\[\Rightarrow A = -1/3, B = -4/9,\]
The ridiculously fabricated example problem

- The template is then $P(t) = At^2 + Bt + C$. Again you don't have exponential and it would be easy to compute:

$$P'' + 2P' - 3P = 2A + 2(2At + B) - 3(At^2 + Bt + C)$$
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$$-3A = 1, \ 4A - 3B = 0, \ 2A + 2B - 3C = 0$$
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$\Rightarrow$ $P(t) = -\frac{1}{3}t^2 - \frac{4}{9}t - \frac{14}{27}$

CHECK YOUR SOLUTION (skip).
And finally you combine all the 4 particular solutions
And finally you combine all the 4 particular solutions together with the complementary solution,
And finally you combine all the 4 particular solutions together with the complementary solution, to get the general solution

\[ y(t) = C_1 e^t + C_2 e^{-3t} \]
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\[ y(t) = C_1e^t + C_2e^{-3t} + \left( \frac{1}{12}t^3 - \frac{1}{16}t^2 + \frac{33}{32}t \right)e^t \]
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\]
Find the general solution to the differential equation
\[ y'' + y = \cos t + t \]
Find the general solution to the differential equation

\[ y'' + y = \cos t + t \]

- The complementary solution

\[ y(t) = c_1 \cos t + c_2 \sin t \]
Quiz Problem 2

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- For the second part of the equation,
Quiz Problem 2

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- For the second part of the equation, namely
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if you are as lazy as me,
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Quiz Problem 2

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- Now we deal the first part,
Quiz Problem 2

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- The template for the first try
Quiz Problem 2

Find the general solution to the differential equation

\[ y'' + y = \cos t + t \]

- The complementary solution is
  \[ y(t) = c_1 \cos t + c_2 \sin t. \]

- For the second part of the equation, namely
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- Now we deal the first part, namely
  \[ y'' + y = \cos t. \]

- The template for the first try is \( P(t) = A \cos t + B \sin t \).
Quiz Problem 2

Find the general solution to the differential equation

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- The complementary solution is

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- For the second part of the equation, namely

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- The template for the first try is \( P(t) = A \cos t + B \sin t \). But this is part of the complementary solution.
Quiz Problem 2

Find the general solution to the differential equation

\[ y'' + y = \cos t + t \]

- The complementary solution is

\[ y(t) = c_1 \cos t + c_2 \sin t. \]

- For the second part of the equation, namely

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- The template for the first try is \( P(t) = A \cos t + B \sin t \). But this is part of the complementary solution. Therefore it is immediate that the first try fails.
Quiz Problem 2

- Multiply your template by another $t$. 
Quiz Problem 2

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$$(t \cos t)' = \cos t - t \sin t,\ (t \sin t)' = \sin t + t \cos t$$

Now let’s get $P, P', P''$: 
Quiz Problem 2

- Multiply your template by another $t$. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$\frac{d}{dt}(t \cos t) = \cos t - t \sin t, \quad \frac{d}{dt}(t \sin t) = \sin t + t \cos t$$

- Now let’s get $P, P', P''$: 

$$P(t) = At \cos t + Bt \sin t,$$
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$$P(t) = At \cos t + Bt \sin t,$$
$$P'(t) = A(\cos t - t \sin t)$$
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$$= A \cos t + B \sin t + Bt \cos t - At \sin t$$
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Now let’s get $P, P', P''$:

\[
\begin{align*}
P(t) &= At \cos t + Bt \sin t, \\
P'(t) &= A(\cos t - t \sin t) + B(\sin t + t \cos t) \\
&= A \cos t + B \sin t + Bt \cos t - At \sin t \\
P''(t) &= -A \sin t
\end{align*}
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Multiply your template by another $t$. Then you have to run into the messy algebra as I did in class. Beforehand, note that

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\[= A \cos t + B \sin t + Bt \cos t - At \sin t\]
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$$= -2A \sin t + 2B \cos t - At \cos t - Bt \sin t.$$
Multiply your template by another $t$. Then you have to run into the messy algebra as I did in class. Beforehand, note that

$$(t \cos t)' = \cos t - t \sin t, (t \sin t)' = \sin t + t \cos t$$

Now let’s get $P, P', P''$:

$$P(t) = A t \cos t + B t \sin t,$$
$$P'(t) = A \cos t - B t \sin t + B \sin t + B t \cos t - A t \sin t,$$
$$P''(t) = -A \sin t + B \cos t + B \cos t - B t \sin t - A \sin t - A t \cos t - B t \sin t,$$
$$P'' + P = -2A \sin t + 2B \cos t - A t \cos t - B t \sin t.$$
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\]
\[
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= A \cos t + B \sin t + Bt \cos t - At \sin t
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= -2A \sin t + 2B \cos t - At \cos t - Bt \sin t.
\]
\[
P'' + P = -2A \sin t + 2B \cos t
\]
Quiz Problem 2

- Compare the coefficients

\[ 2A \sin t + 2B \cos t = \cos t \]

\[ 2A = 0; \quad 2B = 1 \]

\[ A = 0; \quad B = \frac{1}{2} \]

So the particular solution we are looking for is

\[ P(t) = \frac{1}{2} t \sin t \]

So the general solution for the whole ODE is

\[ y(t) = C_1 \cos t + C_2 \sin t + \frac{1}{2} t \sin t + t \]

Remember to check your solution.

Fei Qi (Rutgers University)
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- Remember to check your solution.
It is not impossible to generalize the exponential-shift rule to simplify the computation.
It is not impossible to generalize the exponential-shift rule to simplify the computation. However, the generalization requires a substantial understanding of complex functions and the process of complexification. Considering your workload, I decide not to introduce it here. But anyone who is very interested shall contact me. I'll either teach in person or prepare some additional slides.
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If you use the exponential input theorem in Dr. Mattuck’s lecture,

\[ P(t) = t e^{it} f'(i) = t \cos t + it \sin t. \]

And the particular solution is simply the real part, namely

\[ P(t) = \frac{1}{2} t \sin t. \]

You can actually see the technique issue of complexification; you have to be very clear that when you shall take real part and when you shall take complex part. The slightest confusion or mistake will give a wrong solution. That’s why I don’t want to talk about it here.
Quiz Problem 2: Remarks

- If you use the exponential input theorem in Dr. Mattuck’s lecture, then you immediately get

\[ P(t) = te^{itf'(i)} = t\cos t + it\sin t \]

and the particular solution is simply the real part, namely

\[ P(t) = \frac{1}{2}t\sin t \]

You can actually see the technique issue of complication: you have to be very clear that when you shall take real part and when you shall take complex part. The slightest confusion or mistake will give a wrong solution. That’s why I don’t want to talk about it here.

After several weeks of heavy course load elsewhere, you may forget the right way of doing it.
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\[ \tilde{P}(t) \]
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Homework Problem 3.5.7

Find the general solution of the ODE

\[ y'' + 9y = t^2 e^{3t} + 6 \]
Homework Problem 3.5.7

Find the general solution of the ODE

\[ y'' + 9y = t^2e^{3t} + 6 \]

- The complementary solution

The second term would be easy: Just put in \( P(t) = A \) and by
\[ P'' + 9P = 9A \]
one has \( P(t) = \frac{2}{3} \).

Let's look at the first term. The template for the first try is
\[ P(t) = e^{3t}(At^2 + Bt + C) \]
Homework Problem 3.5.7

Find the general solution of the ODE

\[ y'' + 9y = t^2e^{3t} + 6 \]

The complementary solution is

\[ C_1 \cos 3t + C_2 \sin 3t \]
Find the general solution of the ODE

\[ y'' + 9y = t^2 e^{3t} + 6 \]

- The complementary solution is

\[ C_1 \cos 3t + C_2 \sin 3t \]

- Again separate it.
Find the general solution of the ODE

\[ y'' + 9y = t^2 e^{3t} + 6 \]

- The complementary solution is

\[ C_1 \cos 3t + C_2 \sin 3t \]

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Find the general solution of the ODE

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Homework Problem 3.5.7

Find the general solution of the ODE

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  \[ P(t) = A \] and by \[ P'' + 9P = 9A = 6 \]
Homework Problem 3.5.7

Find the general solution of the ODE

$$y'' + 9y = t^2 e^{3t} + 6$$

- The complementary solution is

$$C_1 \cos 3t + C_2 \sin 3t$$

- Again separate it. The second term would be easy: Just put in $P(t) = A$ and by $P'' + 9P = 9A = 6$ one has $P(t) = 2/3$. 
Homework Problem 3.5.7

Find the general solution of the ODE

\[ y'' + 9y = t^2e^{3t} + 6 \]

- The complementary solution is

\[ C_1 \cos 3t + C_2 \sin 3t \]

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Fei Qi (Rutgers University)
Homework Problem 3.5.7

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- Let's look at the first term. The template for the first try is
  \[ P(t) = e^{3t}(At^2 + Bt + C) \]
Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$. 
Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P$$
Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P = (D^2 + 9)$$
Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

\[ P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C) \]
Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)
\]
\[
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
\]
Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
= e^{3t}
\]
Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)$$

$$= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)$$

$$= e^{3t}(D^2 + 6D + 18)$$
Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C) \\
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)
\]
Use the exponential-shift law to compute $P'' + 9P$.

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P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C) \]
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= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C) \]
\[
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \]
\[
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Use the exponential-shift law to compute $P'' + 9P$.

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\]
\[
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
\]
\[
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)
\]
\[
= e^{3t}(2A)
\]
Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)
= e^{3t}(2A + 6(2At + B))
\]
Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)
\]
\[
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
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\]
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= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)
\]
\[
= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C))
\]
\[
= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)
\]
Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)
= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C))
= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)
\]

Compare coefficients
Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C) \\
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\
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\]

- Compare coefficients

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= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C))
= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)
\]

- Compare coefficients

\[
e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2e^{3t}
\]
Homework Problem 3.5.7

- Use the exponential-shift law to compute \( P'' + 9P \).

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C) \\
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\
= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C)) \\
= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)
\]

- Compare coefficients

\[
e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2 e^{3t} \\
\Rightarrow 18A = 1,
\]
Use the exponential-shift law to compute $P'' + 9P$. 

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C) \\
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C) \\
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C) \\
= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C)) \\
= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)
\]

Compare coefficients

\[
e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C) = t^2 e^{3t} \\
\Rightarrow 18A = 1, 12A + 18B = 0,
\]
Use the exponential-shift law to compute \( P'' + 9P \).

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)
\]

\[
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
\]

\[
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)
\]

\[
= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C))
\]

\[
= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)
\]

Compare coefficients

\[
e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2 e^{3t}
\]

\[
\Rightarrow 18A = 1, 12A + 18B = 0, 2A + 6B + 18C = 0
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Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)$$
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$$= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)$$
$$= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C))$$
$$= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)$$

Compare coefficients

$$e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2 e^{3t}$$
$$\Rightarrow 18A = 1, 12A + 18B = 0, 2A + 6B + 18C = 0$$
$$\Rightarrow A = 1/18,$$
Use the exponential-shift law to compute $P'' + 9P$.

$$P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)$$
$$= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)$$
$$= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)$$
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Compare coefficients

$$e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2 e^{3t}$$
$$\Rightarrow 18A = 1, 12A + 18B = 0, 2A + 6B + 18C = 0$$
$$\Rightarrow A = 1/18, B = -2A/3 = -1/27, $$
Homework Problem 3.5.7

- Use the exponential-shift law to compute $P'' + 9P$.

\[
P'' + 9P = (D^2 + 9)e^{3t}(At^2 + Bt + C)
= e^{3t}((D + 3)^2 + 9)(At^2 + Bt + C)
= e^{3t}(D^2 + 6D + 18)(At^2 + Bt + C)
= e^{3t}(2A + 6(2At + B) + 18(At^2 + Bt + C))
= e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C)
\]

- Compare coefficients

\[
e^{3t}(18At^2 + (12A + 18B)t + 2A + 6B + 18C = t^2 e^{3t}
\Rightarrow 18A = 1, 12A + 18B = 0, 2A + 6B + 18C = 0
\Rightarrow A = 1/18, B = -2A/3 = -1/27, C = -(2A + 6B)/18 = 1/162
\]
So the particular solution we are looking for is

\[ P(t) = e^{3t}(1 + \frac{1}{18}t^2 + \frac{1}{27}t + \frac{1}{162}) \]

Combined with the results above, the general solution is

\[ y(t) = C_1 \cos 3t + C_2 \sin 3t + e^{3t}(1 + \frac{1}{18}t^2 + \frac{1}{27}t + \frac{1}{162}) + \frac{2}{3} \]
So the particular solution we are looking for is

\[ P(t) = e^{3t} \left( \frac{1}{18} t^2 - \frac{1}{27} t + \frac{1}{162} \right) \]
So the particular solution we are looking for is

\[ P(t) = e^{3t}(\frac{1}{18} t^2 - \frac{1}{27} t + \frac{1}{162}) \]

CHECK! (skipped)
Homework Problem 3.5.7

- So the particular solution we are looking for is

\[ P(t) = e^{3t}(\frac{1}{18} t^2 - \frac{1}{27} t + \frac{1}{162}) \]

- CHECK! (skipped)

- Combined with the results above,
So the particular solution we are looking for is

\[ P(t) = e^{3t} \left( \frac{1}{18} t^2 - \frac{1}{27} t + \frac{1}{162} \right) \]

CHECK! (skipped)

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So the particular solution we are looking for is

\[ P(t) = e^{3t}(\frac{1}{18} t^2 - \frac{1}{27} t + \frac{1}{162}) \]

CHECK! (skipped)

Combined with the results above, the general solution is

\[ y(t) = C_1 \cos 3t + C_2 \sin 3t \]
- So the particular solution we are looking for is

\[ P(t) = e^{3t}\left(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162}\right) \]

- CHECK! (skipped)

- Combined with the results above, the general solution is

\[ y(t) = C_1 \cos 3t + C_2 \sin 3t \]
\[ + e^{3t}\left(\frac{1}{18}t^2 - \frac{1}{27}t + \frac{1}{162}\right) \]
So the particular solution we are looking for is

\[ P(t) = e^{3t}(\frac{1}{18} t^2 - \frac{1}{27} t + \frac{1}{162}) \]

CHECK! (skipped)

Combined with the results above, the general solution is

\[ y(t) = C_1 \cos 3t + C_2 \sin 3t + e^{3t}(\frac{1}{18} t^2 - \frac{1}{27} t + \frac{1}{162}) + \frac{2}{3} \]
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]

- This equation is the resonance equation discussed in great detail in MIT Lecture 14.
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it.
Homework Problem 3.5.12

Find the general solution of

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Homework Problem 3.5.12

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- The complementary solution to this ODE
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.

- The complementary solution to this ODE is

  \[ C_1 \cos \omega_0 t + C_2 \sin \omega_0 t \]
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]

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- The complementary solution to this ODE is

  \[ C_1 \cos \omega_0 t + C_2 \sin \omega_0 t \]

- The template for the first try
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]

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- The complementary solution to this ODE is

\[ C_1 \cos \omega_0 t + C_2 \sin \omega_0 t \]

- The template for the first try is

\[ P(t) = A \cos \omega_0 t + B \sin \omega_0 t. \]
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE is

\[ C_1 \cos \omega_0 t + C_2 \sin \omega_0 t \]

- The template for the first try is

\[ P(t) = A \cos \omega_0 t + B \sin \omega_0 t. \]

But this is part of the complementary solution,
Homework Problem 3.5.12

Find the general solution of

\[ y'' + \omega_0^2 y = \cos \omega t \]

- This equation is the resonance equation discussed in great detail in MIT Lecture 14. Please watch the complete analysis for it. I am here just recording my boardwork when teaching Section 10.
- The complementary solution to this ODE is

\[ C_1 \cos \omega_0 t + C_2 \sin \omega_0 t \]

- The template for the first try is

\[ P(t) = A \cos \omega_0 t + B \sin \omega_0 t. \]

But this is part of the complementary solution, therefore the first try fails.
Homework Problem 3.5.12

- Multiply by $t$
Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$
Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$ 

For convenience of use below,
Homework Problem 3.5.12

- Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$ 

For convenience of use below, note that

$$(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t$$
Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$ 

For convenience of use below, note that

$$
(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t \\
(t \sin \omega_0 t)' = \sin \omega_0 t + \omega_0 t \cos \omega_0 t
$$
Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$  

For convenience of use below, note that

$$(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t$$

$$(t \sin \omega_0 t)' = \sin \omega_0 t + \omega_0 t \cos \omega_0 t$$

Now get all the derivatives:
Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$  

For convenience of use below, note that

$$(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t$$

$$(t \sin \omega_0 t)' = \sin \omega_0 t + \omega_0 t \cos \omega_0 t$$

Now get all the derivatives:

$$P'(t) = A(\cos \omega_0 t - \omega_0 t \sin \omega_0 t)$$
Homework Problem 3.5.12

- Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$  

For convenience of use below, note that

$$\begin{align*} 
(t \cos \omega_0 t)' &= \cos \omega_0 t - \omega_0 t \sin \omega_0 t \\
(t \sin \omega_0 t)' &= \sin \omega_0 t + \omega_0 t \cos \omega_0 t 
\end{align*}$$

- Now get all the derivatives:

$$P'(t) = A(\cos \omega_0 t - \omega_0 t \sin \omega_0 t) + B(\sin \omega_0 t + \omega_0 t \cos \omega_0 t)$$
Homework Problem 3.5.12

- Multiply by \( t \) and try

\[
P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.
\]

For convenience of use below, note that

\[
(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t
\]
\[
(t \sin \omega_0 t)' = \sin \omega_0 t + \omega_0 t \cos \omega_0 t
\]

- Now get all the derivatives:

\[
P'(t) = A(cos \omega_0 t - \omega_0 t \sin \omega_0 t) + B(sin \omega_0 t + \omega_0 t \cos \omega_0 t)
\]
\[
= A \cos \omega_0 t + B \sin \omega_0 t + B \omega_0 t \cos \omega_0 t - A \omega_0 t \sin \omega_0 t
\]
Homework Problem 3.5.12

- Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$ 

For convenience of use below, note that

$$(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t$$
$$(t \sin \omega_0 t)' = \sin \omega_0 t + \omega_0 t \cos \omega_0 t$$

- Now get all the derivatives:

$$P'(t) = A(\cos \omega_0 t - \omega_0 t \sin \omega_0 t) + B(\sin \omega_0 t + \omega_0 t \cos \omega_0 t)$$
$$= A \cos \omega_0 t + B \sin \omega_0 t + B \omega_0 t \cos \omega_0 t - A \omega_0 t \sin \omega_0 t$$

$$P''(t) = -A \omega_0 \sin \omega_0 t$$
Homework Problem 3.5.12

- Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$  

For convenience of use below, note that

$$(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t$$

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- Now get all the derivatives:

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$$= A \cos \omega_0 t + B \sin \omega_0 t + B\omega_0 t \cos \omega_0 t - A\omega_0 t \sin \omega_0 t$$

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Homework Problem 3.5.12

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$$P'(t) = A(\cos \omega_0 t - \omega_0 t \sin \omega_0 t) + B(\sin \omega_0 t + \omega_0 t \cos \omega_0 t)$$

$$= A \cos \omega_0 t + B \sin \omega_0 t + B\omega_0 t \cos \omega_0 t - A\omega_0 t \sin \omega_0 t$$

$$P''(t) = -A\omega_0 \sin \omega_0 t + B\omega \cos \omega_0 t$$

$$+ B\omega_0 (\cos \omega_0 t - \omega_0 t \sin \omega_0 t)$$
Homework Problem 3.5.12

Multiply by \( t \) and try

\[
P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.
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For convenience of use below, note that

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Now get all the derivatives:

\[
P'(t) = A(\cos \omega_0 t - \omega_0 t \sin \omega_0 t) + B(\sin \omega_0 t + \omega_0 t \cos \omega_0 t)
\]
\[
= A \cos \omega_0 t + B \sin \omega_0 t + B\omega_0 t \cos \omega_0 t - A\omega_0 t \sin \omega_0 t
\]
\[
P''(t) = -A\omega_0 \sin \omega_0 t + B\omega \cos \omega_0 t
\]
\[
+ B\omega_0 (\cos \omega_0 t - \omega_0 t \sin \omega_0 t) - A\omega (\sin \omega_0 t + \omega_0 t \cos \omega_0 t)
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Homework Problem 3.5.12

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- Now get all the derivatives:

$$P'(t) = A(\cos \omega_0 t - \omega_0 t \sin \omega_0 t) + B(\sin \omega_0 t + \omega_0 t \cos \omega_0 t)$$

$$= A \cos \omega_0 t + B \sin \omega_0 t + B\omega_0 t \cos \omega_0 t - A\omega_0 t \sin \omega_0 t$$

$$P''(t) = -A\omega_0 \sin \omega_0 t + B\omega \cos \omega_0 t$$

$$+ B\omega_0 (\cos \omega_0 t - \omega_0 t \sin \omega_0 t) - A\omega (\sin \omega_0 t + \omega_0 t \cos \omega_0 t)$$

$$= -2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t - A\omega_0^2 \cos \omega_0 t - B\omega_0^2 t \sin \omega_0 t$$
Homework Problem 3.5.12

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$$P'(t) = A(\cos \omega_0 t - \omega_0 t \sin \omega_0 t) + B(\sin \omega_0 t + \omega_0 t \cos \omega_0 t)$$
$$= A \cos \omega_0 t + B \sin \omega_0 t + B \omega_0 t \cos \omega_0 t - A \omega_0 t \sin \omega_0 t$$

$$P''(t) = -A \omega_0 \sin \omega_0 t + B \omega \cos \omega_0 t$$
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So $P'' + \omega_0^2 P$
Multiply by $t$ and try

$$P(t) = At \cos \omega_0 t + Bt \sin \omega_0 t.$$ 

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$$(t \cos \omega_0 t)' = \cos \omega_0 t - \omega_0 t \sin \omega_0 t$$

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$$P''(t) = -A\omega_0 \sin \omega_0 t + B\omega \cos \omega_0 t$$

$$+ B\omega_0 (\cos \omega_0 t - \omega_0 t \sin \omega_0 t) - A\omega (\sin \omega_0 t + \omega_0 t \cos \omega_0 t)$$

$$= -2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t - A\omega_0^2 \cos \omega_0 t - B\omega_0^2 t \sin \omega_0 t$$

So $P'' + \omega_0^2 P = -2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t$. 
Homework Problem 3.5.12

- Compare the coefficients:

\[ A \neq 0 \sin \theta t + 2B \neq 0 \cos \theta t = \cos \theta t \]

\( A = 0; B = 1 = \left(2 \neq 0 \right) \)

So \( P(t) = \frac{1}{2} \neq 0 t \sin \theta t \).

So the general solution of this ODE is

\[ C_1 \cos \theta t + C_2 \sin \theta t + \frac{1}{2} \neq 0 t \sin \theta t : \]
Compare the coefficients:

\[-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t\]
Compare the coefficients:

\[-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t\]
Homework Problem 3.5.12

- Compare the coefficients:

\[-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t\]

\[\Rightarrow A = 0, B = 1/(2\omega_0).\]
Homework Problem 3.5.12

Compare the coefficients:

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\[\Rightarrow A = 0, B = 1/(2\omega_0).\]

So

\[P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t\]
Compare the coefficients:

\[-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t\]

\[\Rightarrow A = 0, \quad B = 1/(2\omega_0).\]

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\[P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t\]

So the general solution of this ODE is

\[C_1 \cos \omega_0 t + C_2 \sin \omega_0 t\]
Homework Problem 3.5.12

- Compare the coefficients:

\[-2A\omega_0 \sin \omega_0 t + 2B\omega_0 \cos \omega_0 t = \cos \omega_0 t\]

\[\Rightarrow A = 0, B = 1/(2\omega_0).\]

So

\[P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t\]

So the general solution of this ODE is

\[C_1 \cos \omega_0 t + C_2 \sin \omega_0 t + \frac{1}{2\omega_0} t \sin \omega_0 t.\]
Again if you use the exponential input theorem,
Again if you use the exponential input theorem, it is immediate that

\[ \tilde{P}(t) = t \cos \omega_0 t + it \sin \omega_0 t \]

and the particular solution is the real part

\[ \tilde{P}(t) = \frac{t}{2} \sin \omega_0 t \]
Again if you use the exponential input theorem, it is immediate that

\[ \tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} \]
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\[
\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t \cos \omega_0 t + it \sin \omega_0 t}{2\omega_0 i}
\]

and the particular solution

When \( \omega_0 = 1 \), you should easily recover the first part of the Quiz Problem.

If the right hand side becomes \( \sin \omega_0 t \), you should then take the imaginary part of \( \tilde{P}(t) \) as your particular solution.
Again if you use the exponential input theorem, it is immediate that

\[ \tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t \cos \omega_0 t + it \sin \omega_0 t}{2\omega_0 i} \]

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Homework Problem 3.5.12: Remarks

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\[ P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t \]
Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t \cos \omega_0 t + it \sin \omega_0 t}{2\omega_0 i}$$

and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t$$

When $\omega_0 = 1$, 

• Again if you use the exponential input theorem, it is immediate that

\[ \tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t \cos \omega_0 t + it \sin \omega_0 t}{2\omega_0 i} \]

and the particular solution is the real part

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and the particular solution is the real part

\[
P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t
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When \(\omega_0 = 1\), you should easily recover the first part of the Quiz Problem.

If the right hand side becomes \(\sin \omega_0 t\),
Again if you use the exponential input theorem, it is immediate that

$$\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t \cos \omega_0 t + it \sin \omega_0 t}{2\omega_0 i}$$

and the particular solution is the real part

$$P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t$$

When $\omega_0 = 1$, you should easily recover the first part of the Quiz Problem.

If the right hand side becomes $\sin \omega_0 t$, you should then take the imaginary part of $\tilde{P}$
Homework Problem 3.5.12: Remarks

- Again if you use the exponential input theorem, it is immediate that

\[
\tilde{P}(t) = \frac{te^{i\omega_0 t}}{2\omega_0 i} = \frac{t \cos \omega_0 t + it \sin \omega_0 t}{2\omega_0 i}
\]

and the particular solution is the real part

\[
P(t) = \frac{1}{2\omega_0} t \sin \omega_0 t
\]

- When \(\omega_0 = 1\), you should easily recover the first part of the Quiz Problem.

- If the right hand side becomes \(\sin \omega_0 t\), you should then take the imaginary part of \(\tilde{P}\) as your particular solution.
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \ y(0) = 1, \ y'(0) = 0 \]
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution

The complementary solution is

\[ C_1 e^t + C_2 te^t \]
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is
  \[ C_1 e^t + C_2 te^t \]
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Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is
  \[ C_1 e^t + C_2 te^t \]
- Again separate it.
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is
  \[ C_1 e^t + C_2 te^t \]
- Again separate it. It is immediate
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is
  \[ C_1 e^t + C_2 te^t \]

- Again separate it. It is immediate that \( P(t) = 4 \) is a solution of
  \[ y'' - 2y' + y = 4. \]
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is

\[ C_1 e^t + C_2 te^t \]

- Again separate it. It is immediate that \( P(t) = 4 \) is a solution of \( y'' - 2y' + y = 4 \). So we just focus on the first term.
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is

  \[ C_1 e^t + C_2 te^t \]

- Again separate it. It is immediate that \( P(t) = 4 \) is a solution of \( y'' - 2y' + y = 4 \). So we just focus on the first term.

- The template for first try
Book Problem 3.5.17

Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is
  \[ C_1 e^t + C_2 te^t \]

- Again separate it. It is immediate that \( P(t) = 4 \) is a solution of \( y'' - 2y' + y = 4 \). So we just focus on the first term.

- The template for first try is \( P(t) = e^t(At + B) \).
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is
  
  \[ C_1 e^t + C_2 te^t \]

- Again separate it. It is immediate that \( P(t) = 4 \) is a solution of \( y'' - 2y' + y = 4 \). So we just focus on the first term.

- The template for first try is \( P(t) = e^t(At + B) \). You should see immediately
Book Problem 3.5.17

Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is

\[ C_1 e^t + C_2 te^t \]

- Again separate it. It is immediate that \( P(t) = 4 \) is a solution of \( y'' - 2y' + y = 4 \). So we just focus on the first term.

- The template for first try is \( P(t) = e^t(At + B) \). You should see immediately that this coincides with the complementary solutions.
Solve the IVP

\[ y'' - 2y' + y = te^t + 4, \quad y(0) = 1, \quad y'(0) = 0 \]

- The complementary solution is

\[ C_1 e^t + C_2 te^t \]

- Again separate it. It is immediate that \( P(t) = 4 \) is a solution of \( y'' - 2y' + y = 4 \). So we just focus on the first term.

- The template for first try is \( P(t) = e^t(At + B) \). You should see immediately that this coincides with the complementary solutions. so the first try fails.
Modify your template
Modify your template as $P(t) = e^t(At^2 + Bt)$. 

There is nothing concerning the text. So the second try fails.
Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute
Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P$$
Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P = (D - 1)^2$$
Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P = (D - 1)^2(e^t(At^2 + Bt))$$
Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P = (D - 1)^2(e^t(At^2 + Bt))$$

$$= e^t(D + 1 - 1)^2(At^2 + Bt)$$
Modify your template as \( P(t) = e^t(At^2 + Bt) \). Use exponential-shift rule to compute

\[
P'' - 2P' + P = (D - 1)^2(e^t(At^2 + Bt))
\]

\[
= e^t(D + 1 - 1)^2(At^2 + Bt)
\]

\[
= e^tD^2(At^2 + Bt)
\]
Modify your template as \( P(t) = e^t(At^2 + Bt) \). Use exponential-shift rule to compute

\[
P'' - 2P' + P = (D - 1)^2(e^t(At^2 + Bt)) \\
= e^t(D + 1 - 1)^2(At^2 + Bt) \\
= e^t D^2(At^2 + Bt) \\
= e^t 2A
\]
Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

\[
P'' - 2P' + P = (D - 1)^2(e^t(At^2 + Bt))
\]
\[
= e^t(D + 1 - 1)^2(At^2 + Bt)
\]
\[
= e^tD^2(At^2 + Bt)
\]
\[
= e^t2A
\]

There is nothing concerning the $te^t$. 
Modify your template as $P(t) = e^t(At^2 + Bt)$. Use exponential-shift rule to compute

$$P'' - 2P' + P = (D - 1)^2(e^t(At^2 + Bt))$$
$$= e^t(D + 1 - 1)^2(At^2 + Bt)$$
$$= e^tD^2(At^2 + Bt)$$
$$= e^t2A$$

There is nothing concerning the $te^t$. So the second try fails.
Modify your template as $P(t) = e^t(At^3 + Bt^2)$. 

Compute as follows 

$P''P' + P = (D_1^2)(e^t(At^3 + Bt^2)) = e^t D_2(At^3 + Bt^2) = e^t(6At + 2B)$

Compare the coefficients 

$e^t(6At + 2B) = te^t(6A) = 1$; $2B = 0$

So $P(t) = 16t^3 e^t$. 

Fei Qi (Rutgers University)

Recitation 7: a. Finding particular solutions
Book Problem 3.5.17

- Modify your template as $P(t) = e^t(At^3 + Bt^2)$. Compute as follows
Modify your template as \( P(t) = e^t(At^3 + Bt^2) \). Compute as follows

\[
P'' - 2P' + P
\]
Modify your template as \( P(t) = e^t(At^3 + Bt^2) \). Compute as follows

\[
P'' - 2P' + P = (D - 1)^2(e^t(At^2 + Bt))
\]
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So

$$P(t) = \frac{1}{6} t^3 e^t$$
So the general solution of the ODE
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\[
y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4
\]
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\[ y(t) = C_1 e^t + C_2 te^t + \frac{1}{6} t^3 e^t + 4 \]

Put in the initial values, one gets the following equations

\[ C_1 + 4 = 1 \]

\[ C_2 = 4 \]

Thus the solution to the IVP is

\[ y(t) = 3e^t + 4te^t + \frac{1}{6} t^3 e^t + 4 \]
So the general solution of the ODE is

\[ y(t) = C_1 e^t + C_2 t e^t + \frac{1}{6} t^3 e^t + 4 \]

Put in the initial values, one gets the following equations

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\[ C_1 + C_2 = 1 \]
So the general solution of the ODE is

\[ y(t) = C_1 e^t + C_2 te^t + \frac{1}{6} t^3 e^t + 4 \]

Put in the initial values, one gets the following equations

\[
C_1 + 4 = 1 \\
C_1 + C_2 = 1
\]

So \( C_1 = -3, \ C_2 = 4 \)
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So the general solution of the ODE is

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So \( C_1 = -3 \), \( C_2 = 4 \) and thus the solution to the IVP is

\[ y(t) = -3e^t + 4te^t + \frac{1}{6} t^3 e^t + 4 \]
The End