FORMULAS

Polynomial of degree $\leq n$ interpolating f at $x_0, \ldots, x_n$

$$\text{Newton form :} \qquad P_n(x) = \sum_{i=0}^{n} f[x_0, x_1, \ldots, x_i] \prod_{j=0}^{i-1} (x - x_j)$$

$$\text{Lagrange form :} \qquad P_n(x) = \sum_{j=0}^{n} L_{j,n}(x) f(x_j), \text{ where } L_{j,n}(x) = \prod_{\substack{i=0 \\ i \neq j}}^{n} \left(\frac{x - x_i}{x_j - x_i} \right).$$

$$\text{Error :} \qquad f(x) - P_n(x) = f[x_0, x_1, \ldots, x_n, x] \prod_{i=0}^{n} (x - x_i) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{i=0}^{n} (x - x_i)$$

where

$$f[x_i, x_{i+1}, \ldots, x_k] = \frac{f[x_{i+1}, , \ldots, x_k] - f[x_i, \ldots, x_{k-1}]}{x_k - x_i}$$

The polynomials $\Phi_0, \Phi_1, \ldots$ defined below are an orthogonal set of polynomials.

$$\Phi_0 = 1, \qquad \Phi_1 = x - \alpha_1, \qquad \Phi_k = x\Phi_{k-1} - \alpha_k \Phi_{k-1} - \beta_k \Phi_{k-2}, \quad k = 2, 3, \ldots$$

where

$$\gamma_k = (\Phi_k, \Phi_k), \quad k = 0, 1, \ldots, \qquad \alpha_k = (x\Phi_{k-1}, \Phi_{k-1})/\gamma_{k-1}, \quad k = 1, 2, \ldots,$$
$$\beta_k = (x\Phi_{k-1}, \Phi_{k-2})/\gamma_{k-2}, \quad k = 2, 3, \ldots.$$

The general $(p+1)$ step Linear Multistep Method is given by:

$$y_{n+1} = \sum_{i=0}^{p} a_i y_{n-i} + h \sum_{i=-1}^{p} b_i f_{n-i}.$$