

Math 477: Mathematical Theory Of Probability

Final Exam

August 16th, 2006

Name:

1. **(20 pts)** Stores A , B and C have 50, 75, and 100 employees and, respectively, 50, 60, and 70 percent of these are women. Resignations are equally likely among all employees, regardless of sex. One employee resigns, and this is a woman. What is the probability that she works in store C ?

2. (a) **(20 pts)** Suppose that the proportion of people in the United States suffering from a rare form of cancer is **0.001**. We take a random sample of 20,000 people. What is the probability that at least 10 but no more than 14 people in our sample have cancer? State **ALL** your assumptions.
- (b) **(20 pts)** Suppose that the proportion of people in the United States suffering from a rare form of cancer is **0.0001**. We take a random sample of 20,000 people. What is the probability that at least 10 but no more than 14 people in our sample have cancer? State **ALL** your assumptions.

3. Suppose that each child born to a couple is equally likely to be a boy or a girl independent of the sex distribution of the other children in the family. For a couple having 5 children, compute the probabilities of the following events:
- (a) **(4 pts)** All children are of the same sex.
 - (b) **(4 pts)** The 3 eldest are boys and the others girls.
 - (c) **(4 pts)** Exactly 3 are boys.
 - (d) **(4 pts)** The 2 oldest are girls.
 - (e) **(4 pts)** There is at least 1 girl.

4. **(20 pts)** Assume the math scores on the SAT test are normally distributed with mean 500 and standard deviation 60, and the verbal scores are normally distributed with mean 450 and standard deviation 80. If two students who took both tests are chosen at random, what is the probability that the first student's math score exceeds the second student's verbal score? State **ALL** your assumptions.

5. **(20 pts)** Suppose A , B and C are mutually independent events with probabilities $\mathbb{P}(A) = 0.5$, $\mathbb{P}(B) = 0.8$ and $\mathbb{P}(C) = 0.3$. What is the probability that at least one of these events occur?

6. Suppose a random variable X has a moment generating function

$$M_X(t) = \left(\frac{2 + e^t}{3}\right)^9$$

- (a) **(10 pts)** Calculate the variance of X .
- (b) **(10 pts)** Find the probability that $X = 0$.

7. Let X and Y be jointly continuous random variables with joint density function

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2} & \text{if } x > 0, y > 0 \text{ and } x + y < 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) **(10 pts)** Check that $f_{X,Y}$ is in fact a joint density function.
- (b) **(10 pts)** Find the marginal density functions of X and Y .
- (c) **(20 pts)** Find $\rho(X, Y)$, the correlation between X and Y .

8. X and Y are discrete random variables whose joint mass function, $p_{X,Y}(x, y)$, is given in the table:

$p_{X,Y}$	x=1	x=2	x=5
y=1	0.12	0.03	0.15
y=2	0.20	0.05	0.25
y=3	0.08	0.02	0.10

- (a) **(5 pts)** Find the marginal mass functions p_X and p_Y .
- (b) **(5 pts)** What is the probability that $Y = 3$ given that $X = 2$?
- (c) **(10 pts)** Are X and Y independent random variables?

9. (**Extra Problem, 20 pts**) If X and Y are independent standard normal random variables, determine the joint density function of (U, V) where

$$U = X \quad V = \frac{X}{Y}$$

Then use your result to find the distribution of the quotient of two independent standard normal random variables.