

Math 421 / Quiz 4 (Solution)

1. If $\mathbf{A}^{-1} = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$, what is \mathbf{A} ?

Sln: Well, \mathbf{A} is the inverse of \mathbf{A}^{-1} . So $\mathbf{A} = \frac{1}{-1} \begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$

2. Is there a matrix $\mathbf{A} \in M_{2 \times 2}(\mathbf{R})$ such that $\mathbf{A}^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$? If your answer is yes, then find it. If your answer is not, then justify why such a matrix does not exist.

Sln: NO. Such a matrix \mathbf{A} can not exist because if $\mathbf{A}^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then by taking determinants (and using the property that $\det(\mathbf{CD}) = \det(\mathbf{C})\det(\mathbf{D})$) we would conclude that $(\det(\mathbf{A}))^2 = -1$, which is impossible.