

**Math 421 / Quiz 3 (Solution)**

1. Let  $\mathbf{A} = \begin{pmatrix} 2 & 1 & 7 \\ 1 & 0 & 2 \\ -1 & 5 & 13 \end{pmatrix}$

- (a) Reduce the matrix  $\mathbf{A}$  to a row-echelon form. Call it  $\mathbf{B}$ .

**Sln:** I got  $\mathbf{B} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

- (b) What is the rank of  $\mathbf{B}$ ?

**Sln:** The rank of  $\mathbf{B}$  is the maximum number of linearly independent rows, therefore the rank of  $\mathbf{B}$  is 2.

- (c) What is the rank of  $\mathbf{A}$ ?

**Sln:** We know that row operations preserve the row space of a matrix, therefore the rank of  $\mathbf{A}$  is 2 also.

- (d) Are the rows of  $\mathbf{A}$  linearly independent? Justify your answer.

**Sln:** Since the rank of  $\mathbf{A}$  is 2, its rows have to be linearly dependent. If they were linearly independent then the rank of  $\mathbf{A}$  would have been 3 instead of 2.

- (e) What about the columns?

**Sln:** We noticed in class that the dimension of the column space is the same as the dimension of the row space of a matrix. Thus the columns are linearly dependent too.