Math 421 / Formula Sheet

Function	Laplace transform
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$e^{at}f(t)$	$F(s-a)$, where $F(s)=\mathcal{L}\{f(t)\}$
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}F(s)$, where $F(s)=\mathcal{L}\{f(t)\}$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$, where $F(s)=\mathcal{L}\{f(t)\}$
$\delta(t-t_0)$	$e^{-t_0 s}$
$f*g$	$F(s)G(s)$, where $F(s)=\mathcal{L}\{f(t)\}$ and $G(s)=\mathcal{L}\{g(t)\}$
f'	$sF(s)-f(0)$, where $F(s)=\mathcal{L}\{f(t)\}$
f	$\int_0^\infty e^{-st}f(t)dt$

Some trig identities

$$\sin a \cos b = \frac{1}{2}\left(\sin(a+b)+\sin(a-b)\right)$$
$$\cos a \cos b = \frac{1}{2}\left(\cos(a+b)+\cos(a-b)\right)$$
$$\sin a \sin b = \frac{1}{2}\left(\cos(a+b)-\cos(a-b)\right)$$
$$\sin^2 x = 1 - cos^2 x = \frac{1-\cos(2x)}{2}$$

Let me remind you that the coefficients of the Fourier series of f on $[-p,p]$

$$\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{p}x\right)+\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{p}x\right)$$

are given by

$$a_n=\frac{1}{p}\int_{-p}^{p} f(x)\cos\left(\frac{n\pi}{p}x\right)dx, \qquad b_n=\frac{1}{p}\int_{-p}^{p} f(x)\sin\left(\frac{n\pi}{p}x\right)dx$$

The solutions to the Sturm-Liouville problem

$$\begin{cases} X''+\lambda X=0 \\ X(0)=0 \\ X(L)=0 \end{cases}$$

are given by $X_n(x)=\sin\left(\frac{n\pi}{L}x\right)$ with $\lambda_n=\frac{n^2\pi^2}{L^2}$ for $n \geq 1$.