Math 421: Advanced Calculus for Engineers

Second Exam

July 5th, 2007

Name:

1. Answer true/false. Justify your answer.
   (a) **(5 pts)** If $\mathbf{A} = 5\mathbf{B}$, then $\det \mathbf{A} = 5 \det \mathbf{B}$.
   (b) **(5 pts)** If 0 is an eigenvalue of $\mathbf{A}$, then $\mathbf{A}$ is singular.

(c) **(5 pts)** The matrix $\mathbf{A} = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix}$ cannot be diagonalized.

(d) **(5 pts)** The product of two symmetric matrices is symmetric.

2. Let $\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$. $\mathbf{A}$ has three different eigenvalues and those are 0, 1 and 2.

   (a) **(10 pts)** Find a basis of $\mathbf{R}^3$ consisting of eigenvectors of $\mathbf{A}$.

   (b) **(10 pts)** Find a diagonal matrix $\mathbf{D}$ and an invertible matrix $\mathbf{P}$ so that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D}$.

(c) **(10 pts)** Find $\mathbf{P}^{-1}$ by the method of cofactors.

(d) **(10 pts)** Compute $\mathbf{A}^6$.

3. **(15 pts)** Let $\mathbf{M} = \begin{pmatrix} a & 0 & 0 & a & a \\ 0 & 0 & b & 0 & 0 \\ c & c & 0 & c & 0 \\ d & 0 & d & d & 0 \\ 0 & e & 0 & e & e \end{pmatrix}$. Compute its determinant.

4. Let $\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}$

(a) **(15 pts)** Compute $\mathbf{A}^{-1}$ by row operations.

(b) **(10 pts)** Solve the linear system $\mathbf{AX} = \mathbf{B}$ where $\mathbf{B} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$

5. **(Extra problem, 5 pts)** Calculate the determinant of $adj(\mathbf{A})$, the adjoint of $\mathbf{A}$, in terms of $\det \mathbf{A}$.

Math 421 / Formula Sheet

Function	Laplace transform
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$e^{at}f(t)$	$F(s-a)$, where $F(s)=\mathcal{L}\{f(t)\}$
$t^n f(t)$	$(-1)^n\frac{d^n}{ds^n}F(s)$, where $F(s)=\mathcal{L}\{f(t)\}$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$, where $F(s)=\mathcal{L}\{f(t)\}$
$\delta(t-t_0)$	$e^{-t_0 s}$
$f*g$	$F(s)G(s)$, where $F(s)=\mathcal{L}\{f(t)\}$ and $G(s)=\mathcal{L}\{g(t)\}$
f'	$sF(s)-f(0)$, where $F(s)=\mathcal{L}\{f(t)\}$
f	$\int_0^\infty e^{-st}f(t)dt$

Useful identities

$$\sin(a+b)=\sin(a)\cos(b)+\sin(b)\cos(a)$$
$$\cos(a+b)=\cos(a)\cos(b)-\sin(a)\sin(b)$$

Let me remind you that...

$adj\mathbf{A}=\mathbf{C}^T$, where $\mathbf{C}=(C_{ij})$ is the matrix of cofactors.

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