Math 421: Advanced Calculus for Engineers

First Exam

June 14th, 2007

Name:

1. Answer true/false. Justify your answer.

   (a) **(5 pts)** The function $(e^t)^{10000}$ is not of exponential order.

   (b) **(5 pts)** $F(s) = \frac{s^2}{s^2+1}$ is not the Laplace transform of a function that is piecewise continuous and of exponential order.

   (c) **(5 pts)** If $\mathcal{L}\{f\} = F(s)$ and $\mathcal{L}\{g\} = G(s)$, then $\mathcal{L}\{fg\} = \mathcal{L}\{f\}\mathcal{L}\{g\}$.

2. Calculate the following Laplace transforms and inverse Laplace transforms:

   (a) **(7.5 pts)** $\mathcal{L}^{-1}\left\{\frac{1}{(2s-1)^4}\right\}$

   (b) **(7.5 pts)** $\mathcal{L}^{-1}\left\{\frac{s+\pi}{s^2+\pi^2}e^{-s}\right\}$

   (c) **(5 pts)** $\mathcal{L}\{te^{8t}\}$

3. **(25 pts)** Use the Laplace transform to solve the linear system:

$$\begin{aligned} \frac{dx}{dt} + y &= t \\ 4x + \frac{dy}{dt} &= 0 \end{aligned}$$

such that $x(0) = 1$ and $y(0) = 2$.

4. **(20 pts)** Let $\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 4 & 5 & 6 \end{pmatrix}$. Calculate the quantities that make sense:

   (a) $\mathbf{AB}$

   (b) $\mathbf{BA}$

   (c) $\mathbf{A^T B^T}$

   (d) $\mathbf{B^T A^T}$

   (e) $((\mathbf{A} + \mathbf{B^T})^\mathbf{T} - \mathbf{A^T})^\mathbf{T} - \mathbf{B^T}$

5. (a) **(10 pts)** Use either Gaussian elimination or Gaussian-Jordan elimination to solve the given system or show that no solution exists

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 2 \\ 3x_1 - x_2 + 2x_3 &= 5 \\ 2x_1 + x_2 + x_3 &= 1 \end{aligned}$$

(b) **(10 pts)** Use your solution to part (a) to get the number of solutions of the following linear system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 0 \\ 3x_1 - x_2 + 2x_3 &= 0 \\ 2x_1 + x_2 + x_3 &= 0 \end{aligned}$$

6. **(Extra problem, 5 pts)** Calculate

$$\int_0^\infty te^{-t}\sin(2t)dt$$

Math 421 / Formula Sheet

Function	Laplace transform
e^{at}	$\frac{1}{s-a}$
t^n	$\frac{n!}{s^{n+1}}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$e^{at}f(t)$	$F(s-a)$, where $F(s)=\mathcal{L}\{f(t)\}$
$t^n f(t)$	$(-1)^n \frac{d^n}{ds^n}F(s)$, where $F(s)=\mathcal{L}\{f(t)\}$
$f(t-a)\mathcal{U}(t-a)$	$e^{-as}F(s)$, where $F(s)=\mathcal{L}\{f(t)\}$
$f*g$	$F(s)G(s)$, where $F(s)=\mathcal{L}\{f(t)\}$ and $G(s)=\mathcal{L}\{g(t)\}$
f'	$sF(s)-f(0)$, where $F(s)=\mathcal{L}\{f(t)\}$
f	$\int_0^\infty e^{-st}f(t)dt$

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