

1. Evaluate the double integral

$$\iint_D y dA$$

where  $D$  is the region in the first quadrant bounded by the parabolas  $y = x^2$  and  $y = 8 - x^2$ .

**Sln:**

Define  $I := \iint_D y dA$ . The first thing to do in a problem like this is to draw the region  $D$ . Please do it and convince yourself that a way to parametrize  $D$  is the following:

$$\begin{aligned} 0 &\leq x \leq 2 \\ x^2 &\leq y \leq 8 - x^2 \end{aligned}$$

Thus,  $I$  can be written as the double integral

$$\begin{aligned} I &= \int_0^2 \int_{x^2}^{8-x^2} y dy dx \\ &= \frac{1}{2} \int_0^2 [(8-x^2)^2 - x^4] dx \\ &= \frac{1}{2} \int_0^2 [64 - 16x^2] dx \\ &= 128/3 \end{aligned}$$