Name:

640:244–03 SPRING 2007 Quiz 9 April 11, 2007

1. Find the general solution of the linear system $\mathbf{x}' = A\mathbf{x}$, where $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ and $A = \begin{pmatrix} 1 & -9 \\ 1 & 1 \end{pmatrix}$. Your answer should be expressed in terms of real functions.

$$(1-\lambda)^2+9=0 \iff (1-\lambda)^2=-9 \iff 1-\lambda=\pm 3i \iff \lambda = 1\pm 3i$$

$\lambda = 1+3i$

$$\begin{pmatrix} -3i & -9 \\ 1 & -3i \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow v_1 - 3iv_2 = 0 \Rightarrow \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 3i \\ 1 \end{pmatrix}$$

$$x_1 = \begin{pmatrix} 3i \\ 1 \end{pmatrix} e^{(1+3i)t} = \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 0 \end{pmatrix} i\right] e^t \cdot (\cos 3t + i \sin(3t))$$

$$= \left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}\cos 3t - \begin{pmatrix} 3 \\ 0 \end{pmatrix}\sin 3t + i\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\sin 3t + \begin{pmatrix} 3 \\ 0 \end{pmatrix}\cos 3t\right)\right] e^t$$

$\vec{x}_3 = \operatorname{Re}\vec{x}_1$, $\vec{x}_4 = \operatorname{Im}\vec{x}_1$

$$\Rightarrow \vec{x}_3 = \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\cos 3t - \begin{pmatrix} 3 \\ 0 \end{pmatrix}\sin 3t\right)e^t; \quad \vec{x}_4 = \left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}\sin 3t + \begin{pmatrix} 3 \\ 0 \end{pmatrix}\cos 3t\right)e^t$$

So

$$\vec{x} = c_1 \begin{pmatrix} -3\sin 3t \\ \cos 3t \end{pmatrix} e^t + c_2 \begin{pmatrix} 3\cos 3t \\ \sin 3t \end{pmatrix} e^t$$

2. Suppose that the 2×2 matrix A has eigenvalues r_1 and r_2. In each case below, give the type of the critical point at the origin for the system $\mathbf{x}' = A\mathbf{x}$: spiral sink, spiral source, nodal sink, nodal source, center, or saddle point.

(a) $r_1 = 2$, $r_2 = 5$. Type: nodal source.

(b) $r_1 = 2i$, $r_2 = -2i$. Type: center.

(c) $r_1 = -1 + i\sqrt{3}$, $r_2 = -1 - i\sqrt{3}$. Type: spiral sink.