Name:______________________________

640:244–03 SPRING 2007 Quiz 8 March 27, 2007

1. (a) Find the reduced row echelon form of the matrix $\begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 3 & 1 & 7 \\ 1 & 1 & 2 & 4 \end{pmatrix}$.

$$\xrightarrow[\tilde{R}_3 = R_3 - R_1]{\tilde{R}_2 = R_2 - 2R_1} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & -1 & 3 & 1 \\ 0 & -1 & 3 & +1 \end{pmatrix} \xrightarrow{\tilde{R}_2 = -R_1} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -3 & -1 \\ 0 & -1 & 3 & 1 \end{pmatrix} \xrightarrow{\tilde{R}_3 = R_3 + R_2} \begin{pmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

(b) Using your result from (a), find all solutions of

$$\left.\begin{aligned} x_1 + 2x_2 - x_3 &= 3 \\ 2x_1 + 3x_2 + x_3 &= 7 \\ x_1 + x_2 + 2x_3 &= 4 \end{aligned}\right\} (*)$$

The linear system (*) has the same slns as

$$\begin{cases} x_1 + 2x_2 - x_3 = 3 \\ x_2 - 3x_3 = -1 \end{cases}$$

Hence $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 5 - 5\alpha \\ -1 + 3\alpha \\ \alpha \end{pmatrix}$ is a solution to (*) for any $\alpha \in \mathbb{R}$, and these are all of them.

2. Find *all* eigenvalues and *one* eigenvector of the matrix $A = \begin{pmatrix} 3 & -3 \\ 4 & -5 \end{pmatrix}$. Note: the eigenvalues are small integers.

$$\det \begin{pmatrix} 3-\lambda & -3 \\ 4 & -5-\lambda \end{pmatrix} = (3-\lambda)(-5-\lambda) + 12 = 0$$
$$\iff \lambda^2 + 2\lambda - 3 = 0$$
$$\iff (\lambda + 3)(\lambda - 1) = 0 \iff \boxed{\lambda = 1, 3}.$$

For $\underline{\lambda = 1}$: Look for $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ such that

$$\begin{pmatrix} 2 & -3 \\ 4 & -6 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow 2v_1 - 3v_2 = 0 \Rightarrow v_1 = \frac{3v_2}{2}.$$

Thus $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \begin{pmatrix} \frac{3}{2}v_2 \\ v_2 \end{pmatrix}$ is an eigen~~value~~ vector for all v_2. For instance,

$v = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ is an eigenvector (associated to $\lambda = 1$).