

1. Solve the IVP:  $x^2 y'' - 6y = 0$ ;  $y(1) = 3$ ,  $y'(1) = 0$ .

Look for solutions of the form  $y = x^r$  ( $\rightarrow y' = r x^{r-1}$ ,  $y'' = r(r-1)x^{r-2}$ )

$$\Rightarrow r(r-1) - 6 = 0$$

$$r^2 - r - 6 = 0 \Rightarrow (r-3)(r+2) = 0 \quad (2)$$

So  $y = c_1 x^3 + c_2 x^{-2}$  ( $y' = 3c_1 x^2 - 2c_2 x^{-3}$ ) (2)

Find  $c_1, c_2$ :

$$\begin{cases} c_1 + c_2 = 3 \\ 3c_1 - 2c_2 = 0 \end{cases}$$

$$\Rightarrow c_1 = \frac{6}{5} \quad c_2 = \frac{9}{5} \quad (1)$$

Thus,  $y = \frac{6}{5} x^3 + \frac{9}{5} x^{-2}$

2. Suppose that  $A = \begin{pmatrix} 1 & 0 \\ 2 & 3 \\ -1 & -3 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 2 & -1 & 0 \\ -1 & 0 & 2 & 5 \\ 1 & 3 & 0 & -2 \end{pmatrix}$

(a) Circle the matrix products which are well defined:  $AB$ ,  $BA^T$ ,  $(A^T A)$ ,  $A^2$ .

(b) Choose one of the matrix products that you have circled and compute it.

$$A^T A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & -3 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 3 \\ -1 & -3 \end{pmatrix} = \begin{pmatrix} 6 & 9 \\ 9 & 18 \end{pmatrix}$$