

Formulas	For $y'' + py' + qy = g$: $u'_1 = -gy_2/W(y_1, y_2)$, $u'_2 = gy_1/W(y_1, y_2)$.
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1. A mass of 1 kg is hung from a spring with spring constant 16 newton/meter. The mass is then released from a position 2 meters above its equilibrium position, with a velocity of 8 meter/sec directed upward. Find the resulting motion and give explicitly its **frequency**, **period**, and **amplitude**, expressing your answers in appropriate units. *There is no friction or damping in the system.*

2. Given that the equation $t^2y'' + ty' - y = 0$ has solutions $y_1(t) = t$ and $y_2(t) = 1/t$, find one solution of the equation $t^2y'' + ty' - y = t^5$. Hint: you may use the formulas above.

$$\left. \begin{array}{l} y_1 = t \\ y_2 = t^{-1} \\ g = t^3 \end{array} \right\} \Rightarrow W_{y_1, y_2} = \begin{vmatrix} t & t^{-1} \\ 1 & -t^{-2} \end{vmatrix} = -2t^{-1} \quad (2)$$

So

$$u_1 = - \int \frac{t^3 \cdot t^{-1}}{-2t^{-1}} dt = \frac{1}{2} \frac{t^4}{4} = \frac{1}{8} t^4 \quad (2)$$

$$u_2 = \int \frac{t^3 \cdot t}{-2t^{-1}} dt = -\frac{1}{2} \frac{t^6}{6} = -\frac{1}{12} t^6 \quad (2)$$

Thus, one solution of $t^2y'' + ty' - y = t^5$ is

$$y = \left(\frac{1}{8}t^4\right)(t) - \frac{1}{12}t^6(t^{-1}) = \left(\frac{1}{8} - \frac{1}{12}\right)t^5 = \frac{1}{24}t^5 \quad (2)$$