Name:

640:244-01 SPRING 2007 Quiz 5 February 28, 2007

1. Solve the initial value problem $y'' - 2y' - 3y = e^{2t} + \sin t$, $y(0) = 2$, $y'(0) = 1$.

(i) Solve the homogeneous O.D.E. first:

$$y'' - 2y' - 3y = 0$$
$$r^2 - 2r - 3 = 0$$
$$(r-3)(r+1) = 0$$
$$\Rightarrow \quad y_H = c_1 e^{3t} + c_2 e^{-t}$$

(ii) Find particular solutions:

$Y_1 = Ae^{2t}$:
$$4Ae^{2t} - 4Ae^{2t} - 3Ae^{2t} = e^{2t} \Rightarrow A = -\frac{1}{3}$$

$Y_2 = A\sin t + B\cos t$: $\left(Y_2' = A\cos t - B\sin t,\ Y_2'' = -A\sin t - B\cos t\right)$

$$-A\sin t - B\cos t - 2(A\cos t - B\sin t) - 3(A\sin t + B\cos t) = \sin t$$
$$\sin t(-A + 2B - 3A) + \cos t(-B \mp 2A - 3B) = \sin t$$
$$\sin t(2B - 4A) + \cos t(-2A - 4B) = \sin t$$
$$\begin{cases} 2B - 4A = 1 \\ -2A - 4B = 0 \end{cases} \Leftrightarrow \begin{cases} A = -\frac{2}{10} = -\frac{1}{5} \\ B = \frac{1}{10} \end{cases}$$

(iii) Build general solution:
$$y = c_1 e^{3t} + c_2 e^{-t} - \frac{1}{3}e^{2t} - \frac{1}{5}\sin t + \frac{1}{10}\cos t$$

(iv) Find c_1 & c_2 to satisfy initial conditions:
$$\begin{matrix} y(0) = 2 \Rightarrow \\ y'(0) = 1 \Rightarrow \end{matrix} \begin{cases} 2 = c_1 + c_2 - \frac{1}{3} + \frac{1}{10} \\ 1 = 3c_1 - c_2 - \frac{2}{3} - \frac{1}{5} \end{cases} \Rightarrow \begin{cases} c_1 = \frac{41}{40} \\ c_2 = \frac{147}{120} \end{cases}$$

(v) Solution: $y = \frac{41}{40}e^{3t} + \frac{147}{120}e^{-t} - \frac{1}{3}e^{2t} - \frac{1}{5}\sin t + \frac{1}{10}\cos t$