Name:______________________________

640:244–01 SPRING 2007 Quiz 4 February 14, 2007

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1. Solve the initial value problem $y'' - 4y' - 5y = 0, \quad y(0) = 1, \quad y'(0) = -3.$

Look for solutions of the form $y = e^{rt}$:

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$$r^2 - 4r - 5 = 0$$
$$\Rightarrow (r-5)(r+1) = 0$$
$$\Rightarrow r_1 = 5, \; r_2 = -1$$

The general sln to the ODE (homogeneous & linear) is then

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$$y = c_1 e^{5t} + c_2 e^{-t} \quad (y' = 5c_1 e^{5t} - c_2 e^{-t})$$

1

Find c_1, c_2: $\left\{\begin{array}{l} 1 = c_1 + c_2 \\ -3 = 5c_1 - c_2 \end{array}\right\} \Rightarrow c_1 = -\frac{1}{3}, \; c_2 = \frac{4}{3}$

So $y = -\frac{1}{3}e^{5t} + \frac{4}{3}e^{-t}$ is the solution.

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2. A certain numerical method, similar to those we have studied, is used to approximate the value $y(1)$ of solution of some specific problem $y' = f(t, y)$, $y(0) = y_0$ (with known solution). The error in the computation for various step sizes h is as given in the table:

h	0.08	0.04	0.02
Error	0.1906	0.0502	0.0126

2.5

(a) If the computation were carried out with step size $h = 0.01$, approximately what would the error be?

We observe that the error decreases by a factor of 4 (approx) as the step size is cut in half. So when $h = 0.01$ we expect the error to be $\frac{0.0126}{4} = 0.00315$

1.25

(b) Fill in the blank: the global error in the method is order h^p with $p =$ 2

$\frac{\text{Error 1}}{\text{Error 2}} = \left(\frac{h_1}{h_2}\right)^p \xRightarrow{\text{from data}} p = 2$

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(c) Fill in the blank: the local truncation error in the method is order h^q with $q =$ 3

One more than the p for global error