Name:____________________

640:244–02 SPRING 2007 Quiz 1 January 24, 2007

1. Solve for $y(t)$: $y' + \frac{2}{t}y = -1, \quad y(1) = 1.$

This is a first order linear O.D.E. $y' + p(t)y = q(t)$ where $p(t) = \frac{2}{t}$ and $q(t) = -1$. Thus, with $\mu(t) = e^{\int p\,dt} = t^2$

$$y(t) = \frac{\int \mu q\,dt + c}{\mu} = \frac{\int 2t^2 dt + c}{t^2} = \frac{\frac{2}{3}t^3 + c}{t^2} = \frac{2}{3}t + \frac{c}{t^2}$$

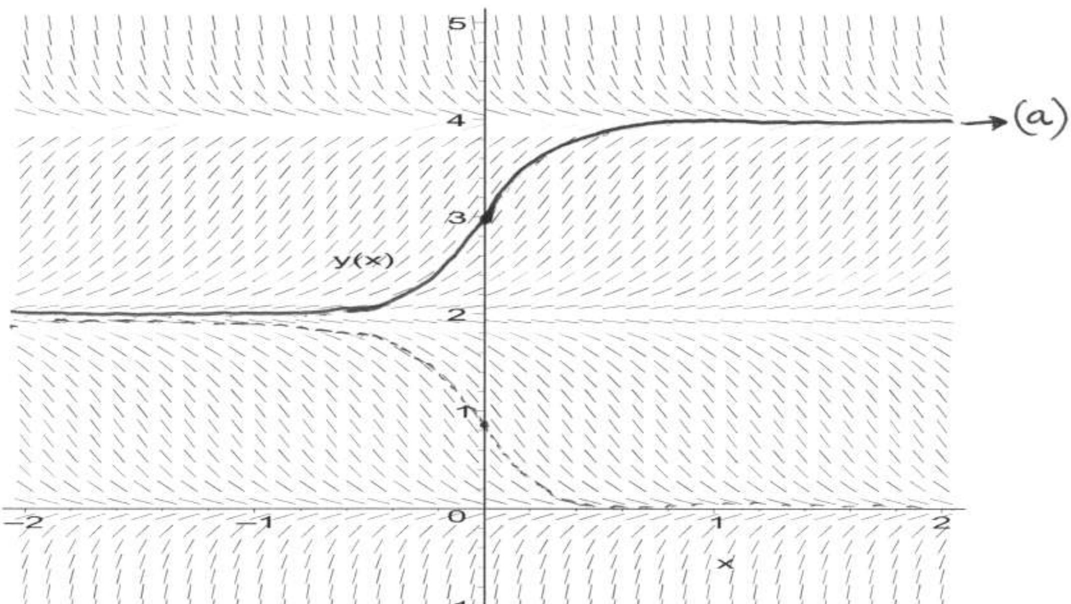
But $y(1) = 1$. So

$$1 = \frac{2}{3} + C \Rightarrow C = -\frac{1}{3}$$

Therefore,

$$y(t) = \frac{2}{3}t - \frac{1}{3t^2}$$

2. Here is the direction field of a certain differential equation:



(a) On the figure, sketch the solution which satisfies $y(0) = 3$.

(b) Find a value of y_0 such that if $y(x)$ is a solution with $y(0) = y_0$ then $\lim_{x\to\infty} y(x) = 0$ and $\lim_{x\to-\infty} y(x) = 2$: $y_0 =$ _1_. or any number in $(0, 2)$