Name:

640:244-02 SPRING 2007 | Quiz 10 | April 18, 2007

1. Consider the system of equations

$$\begin{aligned} x'(t) &= x\,, \\ y'(t) &= 9 + x - y^2\,. \end{aligned}$$

(a) Find all critical points of this system.

$$\left\{\begin{aligned} x'(t) &= x &= 0 \\ y'(t) &= 9+x-y^2 &= 0 \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} x &= 0 \\ y &= \pm 3 \end{aligned}\right\} \Rightarrow \boxed{\left\{\begin{matrix}(0,3) \\ (0,-3\end{matrix}\right\}}$$

(b) Choose *one* of the critical points found in (a). Linearize the above system near this point, that is, find a *linear* system that is a good approximation to the true system near this point.

I choose $(0,3)$. Let $F(x,y) = x$, $G(x,y) = 9 + x - y^2$. The linear system that is a good approximation near $(0,3)$ is

$$\vec{X}' = A\vec{X} \quad \text{where} \quad A = \begin{pmatrix} F_x(0,3) & F_y(0,3) \\ G_x(0,3) & G_y(0,3) \end{pmatrix}$$

Since $F_x = 1$, $F_y = 0$, $G_x = 1$, $G_y = -2y$ then $A = \begin{pmatrix} 1 & 0 \\ 1 & -6 \end{pmatrix}$

$$\boxed{\vec{X}' = \begin{pmatrix} 1 & 0 \\ 1 & -6 \end{pmatrix}\vec{X}}$$

(c) Find the type and stability of the critical point studied in (b).

Eigenvalues of A.

$$\begin{vmatrix} 1-\lambda & 0 \\ 1 & -6-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)(-6-\lambda) = 0 \Rightarrow \lambda = 1, -6$$

Thus (0.3) is a saddle point and therefore unstable.