Homework for Math 300; Fall 2006.

Due Mon Nov 20 in class:

Definion of Rational Numbers, Addition, Multiplication, and Less than.

Define an equivalence relation $\sim$ on $\mathbb{Z}\times(\mathbb{Z}-\{0\})$ by $(a,b)\sim(c,d) \iff ad=bc$. (Think of (a,b) as a/b.)

Define $\mathbb{Q}=(Z\times(\mathbb{Z}-\{0\}))/\sim$. We write the equivalence class of (a,b) as $[a,b]$.

Addition is $\mathbb{Q}\times\mathbb{Q}\to\mathbb{Q}$, $[a,b]+[c,d]=[ad+bc,bd]$. [common denominators].

Multiplication is $\mathbb{Q}\times\mathbb{Q}\to\mathbb{Q}$, $[a,b][c,d]=[ac,bd]$.

We define non-negative rational numbers by $[a,b]\geq 0$ iff $ab\geq 0$.

We say $[a,b]\geq[c,d]$ iff $[a,b]-[c,d]\geq 0$.

If $[a,b]$ is non-zero, we define inverses by $[a,b]^{-1}=[b,a]$.

Division of rational numbers is defined by $p/q=pq^{-1}$.

An integer is aa rational number that is equivalent to $[n,1]$ for some n. (Technically, this is a redefinition of an integers as a special kind of rational number.)

Problem: Prove that addition of rational numbers is well-defined, that is $[a,b]+[c,d]=[a',b']+[c',d']$ if $[a,b]=[a',b']$ and $[c,d]=[c',d']$.

Problem: Prove that if p,q are rational numbers and $p<q$ then $p<(p+q)/2<q$.

Problem: Prove that any rational number q has a unique representative a/b so that b is coprime to a.

Problem: Prove that the set of rational numbers of the form $1/n, n\in N-\{0\}$ has no smallest element.

Problem: Prove that $\sqrt{6}$ is irrational. (This is abuse of language since we haven't introduced irrational numbers yet. What it really means is, show there is no rational number q that is a square root of 6.)

Problem: Prove that for any rational number x, $\sqrt{6}+x$ is irrational.

Extra Credit: Prove that $\sqrt{2}+\sqrt{3}$ is irrational.