Homework 6 for Math 300; Fall 2006.

Due Thurs Oct 19 in class:

Problems Section 3.1, # 3,5,8,9(a,e,f), 11(c,e) 14

Problems Section 3.2, # 1, 4(a,e,c) 9,10,12

Problem Prove that $((A \subseteq B) \wedge (C \subseteq D)) \Rightarrow (A \times C) \subseteq (B \times D)$.

Problem Prove that if R, S are relations in $A \times B$ and $B \times C$ respectively then $dom(R \circ S) \subseteq dom(R)$ and $rng(R \circ S) \subseteq rng(S)$.

Problem Prove that $(A = \emptyset \vee B = \emptyset) \iff (A \times B) = \emptyset$.

Rules: In addition to the symbols before, you may also used ordered pairs. In addition to the rules before, you may use the definitions
$\{(a,b)|P(a,b)\} = \{x|\exists a, b, x = (a,b) \wedge P(a,b)\}$. (set def prop for ordered pairs.)
For example, $x \in \{(a,b)|a \in A \wedge b \in B\} \iff \exists a, b, a \in A \wedge b \in B \wedge x = (a,b)$.
$A \times B = \{(a,b)|a \in A \wedge b \in B\}$. (def of product)
$R^{-1} = \{(b,a)|(a,b) \in R\}$. (def of inverse)
$R \circ S = \{(a,c)|\exists b, (a,b) \in R \wedge (b,c) \in S\}$. (def of composition)
$dom(R) = \{a, \exists b, (a,b) \in R\}$ (def of domain)
$rng(R) = \{b, \exists a, (a,b) \in R\}$ (def of domain)

Note: Please make all proofs two-column proofs, not the paragraph proofs used in the book. The left-hand column should contain no English. Condensing several lines into one is acceptable, as long you indicate you are skipping steps on that line. For example, the complete proof of

$x \in A \cup B \vdash x \in A \vee x \in B$

is

(1) $x \in A \cup B$ (hyp)
(2) $A \cup B = \{x|x \in A \vee x \in B\}$ (def of union)
(3) $(A \cup B = \{x|x \in A \vee x \in B\}) \iff (x \in A \cup B \iff x \in \{x|x \in A \vee x \in B\})$ (def of set equality)
(4) $x \in A \cup B \iff x \in \{x|x \in A \vee x \in B\}$ (modus ponens on (2),(3))
(5) $x \in \{x|x \in A \vee x \in B\}$ (modus ponens on (1),(4))
(6) $x \in \{x|x \in A \vee x \in B\} \iff x \in A \vee x \in B$ (set def axiom)
(7) $x \in A \vee x \in B$ (modus ponens on (5),(6))

Now if you want to abbreviate this to

(1) $x \in A \cup B$ (hyp)
(2) $x \in A \vee x \in B$ (def of union, etc.)

you should at least write something like etc. to indicate you are skipping steps. Please only skip steps if it is clear how to get from (1) to (2).