Homework 5 for Math 300; Fall 2006.

Due Thurs Oct 12 in class:

Problem 1: Give hypothesis-free demonstrations of the following statements about sets $A, B, ...$

(a) $A \cup B = B \cup A$
(b) $A \cup (B \cup C) = (A \cup B) \cup C$
(c) $\forall y, y \notin \emptyset$
(d) $A \neq \emptyset \Rightarrow \exists y, y \in A$
(e) $A \cup \emptyset = A$
(f) $(A \subset B) \Rightarrow (A \cup C \subset B \cup C)$
(g) $(A - B) \cup (A \cap B) = A$
(h) $(A \cap B) \subset (A \cup B)$
(i) $(A \cup B) \cap (A \cup C) = A \cup (B \cap C)$

List of Rules: In addition to the logical rules before,
$A = B \iff (\forall x, x \in A \iff x \in B)$ (Definition of Set Equality)
$A \subset B \iff (\forall x, x \in A \Rightarrow x \in B)$ (Definition of Subset)
$A \cap B = \{x | x \in A \wedge x \in B\}$ (Definition of intersection)
$A \cup B = \{x | x \in A \vee x \in B\}$ (Definition of union)
$\emptyset = \{x \neq x\}$ (Definition of Empty Set)
$x = y, P(x) \vdash P(y)$ (Substitution)
$\forall x, P(x) \vdash P(y)$ (Universal instantiation)
$P(y) \vdash \exists x, P(x)$ (Existential generalization)
$\exists x, P(x) \vdash P(y)$ (Existential instantiation, y a restricted (particular) value of x)
$P(y) \vdash \forall x, P(x)$ (Universal generalization, y an unrestricted (general) value of x)

Example Proof that $A \cap B = B \cap A$ is

(1) $x \in A \cap B \iff (x \in A \wedge x \in B)$ (Def of Int)
(2) $(x \in A \wedge x \in B) \iff (x \in B \wedge x \in A)$ (Taut)
(3) $(x \in B \wedge x \in A) \iff x \in B \cap A$ (Def of Int)
(4) $x \in A \cap B \iff x \in B \cap A$ (TB on (1)(2)(3))
(5) $\forall x(x \in A \cap B \iff x \in B \cap A)$ (UG on (4))
(6) $(A \cap B = B \cap A) \iff (\forall x(x \in A \cap B \iff x \in B \cap A))$(Def of Set Equality)
(7) $A \cap B = B \cap A$ (MP on (5), (6))