

Homework 4 for Math 300; Fall 2006.

Due Thurs Sep 28 in class:

Problem 1: (Using inference by cases, deduction theorem,)

Show that $[P \iff S], [Q \iff T] \vdash [[P \wedge Q] \iff [S \wedge T]]$.

Show that $[[\sim P] \iff Q], [Q \iff R], R \vdash [\sim P]$

Show that $[Q \iff R], [[\sim P] \iff Q], [\sim R] \vdash P$

Show that $[Q \iff [[\sim R] \wedge S]], [[S \vee [\sim T]] \iff R], Q \vdash T$

Show that $[P \Rightarrow R], [Q \Rightarrow T], [[R \wedge T] \iff S] \vdash [[P \wedge Q] \Rightarrow S]$.

Show that $[R \Rightarrow T], [[\sim T] \iff S], [[R \wedge [\sim S]] \Rightarrow [\sim Q]] \vdash [R \Rightarrow [\sim Q]]$

Problem 2: In this problem, your task is to determine, for each of the following five claims, whether or not it is a logical consequence of the following three assumptions. Assumptions: 1. If Jim's arm hurts, he pitches badly. 2. If Jim pitches badly, the team loses. 3. If Jim pitches badly or the team loses, the crowd does not applaud. Claims: 1. If the crowd applauds, then Jim's arm does not hurt. 2. If the team loses, Jim's arm hurts. 3. If the crowd does not applaud, then the team loses. 4. If Jim pitches badly, then his arm hurts and the crowd applauds. 5. If Jim's arm does not hurt and he pitches badly and the crowd applauds, then the team loses.

Problem 3: (Using the deduction theorem, proof by contradiction,)

Show that $[[\sim P] \Rightarrow [Q \wedge R]], [P \Rightarrow [\sim S]] \vdash [S \Rightarrow Q]$.

Show that $[[\sim R] \Rightarrow P], [[P \wedge [\sim S]] \Rightarrow Q] \vdash [[\sim[R \vee S]] \Rightarrow Q]$.

Show that $[P \Rightarrow Q], [Q \Rightarrow [\sim R]], R, [S \Rightarrow P] \vdash [\sim S]$.

Show that $[P \Rightarrow Q], [Q \Rightarrow R], [[P \Rightarrow R] \Rightarrow [\sim S]], [[\sim S] \Rightarrow T] \vdash T$.

Show that $[R \Rightarrow S], [[R \Rightarrow [\sim P]] \Rightarrow Q], [S \Rightarrow [\sim P]], T \vdash [Q \wedge T]$.

Show that $[P \vee Q], [[\sim R] \Rightarrow [\sim S]], [[Q \vee P] \Rightarrow S], [[\sim[T \wedge U]] \Rightarrow [\sim R]] \vdash [U \wedge T]$

Show that $[\sim P], [\sim Q], [R \Rightarrow P] \vdash [\sim[Q \vee R]]$.

Show that $[[Q \wedge R] \Rightarrow S], [[\sim S] \wedge R] \vdash [\sim Q]$.

Show that $[P \Rightarrow [[R \wedge T] \Rightarrow S]], R, [S \Rightarrow Q], [\sim[T \Rightarrow Q]] \vdash [\sim P]$.

Show that $[\sim P], [S \Rightarrow [P \vee Q]], [Q \Rightarrow R], [[[S \Rightarrow Q] \wedge T] \Rightarrow U] \vdash [[S \Rightarrow Q] \iff [T \Rightarrow U]]$.

Show that $[P \Rightarrow [\sim Q]], [[\sim R] \Rightarrow Q] \vdash [P \Rightarrow R]$.

Show that $[R \Rightarrow [\sim Q]], [\sim[S \wedge [\sim R]]] \vdash [Q \Rightarrow [\sim S]]$.

Show that $[[\sim P] \Rightarrow [\sim S]], [P \Rightarrow R], [R \Rightarrow [\sim T]] \vdash [S \Rightarrow [\sim T]]$.