Math 251

Practice Midterm 1 Spring Fall 2004

PROBLEM 1 (a) Find the cross product of the vectors $(6,-2,-3)$ and $(1,1,1)$. (b) Find the angle between the same vectors, $(6,-2,-3)$ and $(1,1,1)$, using your answer from part (a) (c) Find the area of the triangle with vertices $(0,0,0),(1,-1,1),(4,3,7)$.

PROBLEM 2 (a) Find the parametric equation for the line L that is the intersection of the planes $2x-3y+5z+3=0, \quad x+z+2=0$. (b) Find the symmetric equations for the line L of part (a).

PROBLEM 3 (a) Draw a picture of the space curve with parametric equation $r(t)=(\sqrt{3}\cos(t),\sqrt{2}\sin(t),\sin(t))$. (b) Using the formula for arclength, find the length of the curve for $t\in[0,T]$. (c) Find the parametric equation for the line tangent to the curve $r(t)$ at $t=\pi/6$. Give an exact answer, not a decimal approximation.

PROBLEM 4 (a) Let $f(x,y)=\frac{x^2-y^2}{x-y}$ if $x\neq y$ $f(x,y)=0$ if $x=y$. Is $f(x,y)$ continuous? Explain your answer. (b) Sketch three level sets of this function. Label each level set with its value.

PROBLEM 5 (a) Find the first partial derivatives of the function $f(x,y)=\log_x(y)$. (b) Find the equation for the tangent plane to $f(x,y)$ at $(x_0,y_0)=(2,1)$. (c) In which direction is the function increasing the most rapidly at the point $(2,1)$? (d) Find the equation for the tangent line to the level set for f at $(x_0,y_0)=(2,1)$.

PROBLEM 6 (a) Prove that $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})=-\mathbf{v}\cdot(\mathbf{u}\times\mathbf{w})$. (b) Prove that if $|\mathbf{v}(t)|=1$ for all t, then $\mathbf{v}(t)$ is perpendicular to $\mathbf{v}'(t)$.

PROBLEM 7 Which of the following are possible paths for an object travelling in the gravitational influence of the sun? (a) ellipse with the sun at center (b) ellipse with the sun at one focus (c) circle with sun at center (d) parabola with sun outside (e) parabola with sun inside (f) hyperbola with sun outside (g) hyperbola with sun inside

PROBLEM 8 Classify the critical points for the function $6x-2x^3-4y^2+2y^4$ as local minima, local maxima, or neither.