Practice Midterm 2 Fall 2006

PROBLEM 1 Find a basis for the subspace of R^4 defined by $x_1+2x_3+x_4=0$.

PROBLEM 2 (a) Find the determinant for $A=\begin{bmatrix} 0 & 2 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ by expanding along the second row.
(b) Find the determinant of A, using properties of the determinant under row-reduction.

PROBLEM 3 True or false: Justify your answer in one or two sentences.
(a) If an invertible square matrix A is diagonalizable, then so is A^{-1}. (b) Suppose A is a square matrix with row vectors v_1, v_2, v_3. If $det(A)=2$, then the determinant of the matrix A' with row vectors $v_1+v_3, v_2, v_1+v_2+v_3$ is also 2. (c) The dimension of the column space of A is the same as the dimension of the row-space. (d) If A is a 7×3 matrix whose rank is 3, then the rows of A are linearly independent. (e) If $Q=Q^{-1}$, then $det(Q)=\pm 1$. (f) The row space is the orthogonal complement of the nullspace. (g) For any square matrix A, $det(-A)=-det(A)$.

PROBLEM 4 Find the eigenvalues and eigenvectors for the matrix $A=\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ Find the diagonalization of A, if it exists.

(b) Same for $A=\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ using complex numbers.

PROBLEM 5

A party is going on in two rooms, the living room and the kitchen, in someone's apartment. Every hour, fifty percent of the people in the living room move to the kitchen, and twenty percent of the people in the kitchen move to the living room.

(a) Find the transition matrix for this problem, that is, the matrix A such that

$$x(t+1)=Ax(t), \qquad where\ x(t)=l(t)k(t)$$

and $l(t), k(t)$ are the number of people in the living room, kitchen at time t.
(b) Find the eigenvectors and eigenvalues for A.
(c) Find matrices S and D such that $A=SDS^{-1}$.
(d) Suppose that initially, 100 party-goers are distributed equally among the two rooms. Find a formula for the number of people in the kitchen at time t.
(e) How many people are in the kitchen, in the limit $t\to\infty$?
(f) Show the evolution of the system on the graph with axes l, k. (t is not an axis!)

PROBLEM 5' (like Problem 5, for practice)
Two companies are competing for customers. Each year, the first loses 25 percent of its customers to the second, while the second loses half.
(a) Write down the state vector and time evolution matrix for this system. That is, represent the system in the form $x(t+1)=Ax(t)$, for some matrix A.
(b) Find the diagonalization of the matrix A.
(c) Suppose that initially, 100 customers are with the first company, and none with the others. Find a formula for the number of customers with the first as a function of the year t.
(d) How many customers does the first company have, for t very large?

PROBLEM 7 Define a sequence $f(n)$ by $f(n+1)=f(n)+f(n-1)$. Find starting values $f(1), f(2)$ so that the limit is zero.

PROBLEM 8
(a) Show that 0 is an eigenvalue of A, if and only if $\det(A)=0$.
(b) Show that the eigenvalues of $I+A$ are the eigenvalues of A plus 1.